



Numerical Analysis of Energy Expenditure for Co-Flow Wall Jet Separation Control

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Abstract

This paper numerically investigates the energy expenditure of the co-flow wall jet for separation control by analyzing the widely studied NASA hump with 2D unsteady Reynolds averaged Navier-Stokes equations. Co-flow wall jet is shown to be effective in both adverse and favorable pressure gradients, but is more efficient in adverse pressure gradients due to lower velocity, lower entropy increase, and more enhanced mixing. An energy-efficient way to devise a co-flow wall jet flow control is two-fold: 1) place the injection near the separation onset point at a slightly downstream location; 2) place the suction slot further downstream with sufficiently long distance in adverse pressure gradient region for a thorough mixing and energy transfer. The vanishing of the counter-clockwise wall jet vorticity appears to indicate a sufficient mixing distance. In that case, the injection plays the dominant role. The suction makes a small contribution with a weak coupling effect, but primarily serves as the flow source for the mass conservation of the zero-net-mass-flux flow control, which is essential to be energy efficient. The co-flow wall jet is also compared with the injection-only and suction-only flow controls, which are effective if the slots are placed at the desirable position slightly downstream of the separation onset point. But if they are not placed near the separation onset locations, the co-flow wall jet is much more efficient and effective than the injection-only or suction-only flow controls due to the coupling effect between the injection and suction.

Nomenclature

AFC	Active flow control
AoA	Angle of attack
APG	Adverse pressure gradient
BC	Boundary condition
c	The hump chord length
C_E	Energy coefficient, $\dot{m}_j U_j^2 / (0.5 q_{ref} U_{ref} S)$
C_L	Lift coefficient
C_μ	Jet momentum coefficient, $\dot{m}_j U_j / (q_{ref} S)$
C_p	Pressure coefficient
$CFWJ$	Co-flow wall jet
$FASIP$	Flow-acoustics-structure interaction package

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FPG	Favorable pressure gradient
h	Slot height
H_t	Total enthalpy
LE	Leading edge
\dot{m}	Mass flow rate, $\rho U A$
\bar{m}	Normalized mass flow rate, $\frac{\dot{m}}{\rho_{ref} U_{ref} A_{ref}}$
Ma	Mach number
P	CFWJ pumping power
P_c	Power coefficient
P_t	Total pressure
q_{ref}	Freestream dynamic pressure, $0.5\rho_{ref}U_{ref}^2$
Re	Reynolds number
PR	Power required
T_t	Total temperature
$URANS$	Unsteady-Reynolds-averaged Navier-Stokes
U_j	Injection jet velocity
U_{ref}	Reference velocity at inlet
U_∞	Freestream velocity away from wall
VP	Velocity profile
$ZNMF$	Zero-net mass flux

Greek symbols

α	Angle of attack
β	Sideslip angle
δ^*	Displacement thickness
Γ	Total pressure ratio
γ	Air specific heat ratio
η	CFWJ pumping system efficiency
ρ_{ref}	Reference density
θ	Momentum thickness
τ	Shear stress

Subscript

c	Corrected
j	Jet
t	Total value
ref	Reference parameters at hump inlet
∞	Freestream flow away from the wall

1 Introduction

Active Flow Control (AFC) has the potential to break through conventional fluid mechanics limitations and provides significant performance improvement to fluid systems [1]. AFC is to transfer external energy to the controlled flow in order to improve the performance of the flow system. Prandtl's rotating cylinder experiment in 1934 was one of the earliest AFC studies to transfer mechanical energy to flow via the surface shear stress of a rotating cylinder [2]. However, systematic studies of AFC have only occurred recently due to more and more challenging applications of flow systems.

For all AFC systems, there are three measures of merit (MoM): 1) effectiveness; 2) power required (PR); and 3) power conversion efficiency (PCE). Effectiveness quantifies the performance enhancement, e.g., removal of flow separation, drag reduction, lift increase, stall prevention, noise mitigation, etc. Power required quantifies the AFC power needed to achieve the targeted effectiveness. Power conversion efficiency quantifies the efficiency to convert the external power (e.g., mechanical, electric, chemical) to the power required by the controlled flow. It determines how much total power will be consumed by the actual flow control system.

For an AFC to benefit industry realistic applications, all the three MoM matter. The ultimate criterion for an AFC is that the system efficiency gain should be greater than the AFC energy expenditure. However, the current AFCs do not always have clear quantification of all the MoM, in particular for the PR and PCE, partially because they may not always be easy to measure. For a zero-net-mass-flux (ZNMF) flow control, it is more straightforward to measure the PR and PCE because it is a closed system. For a non-ZNMF flow control such as the one using injection-only, namely an open system, the PR is difficult to measure because it depends on the path that the mass flow source is supplied. The PCE measurement could be even more difficult because it also depends on the actuators used and their integration with the system (e.g. aircraft). Similarly, for the flow control with suction-only, the PR and PCE depend on the path that the flow is discharged to the sink. In other words, the PR and PCE of the non-ZNMF flow control are path-dependent. A long path, in general, will suffer from a high energy loss, in particular if the path requires a flow turning perpendicular to the controlled flow plane.

Synthetic jets [3–8] generated by the periodic motion of a piston or diaphragm and plasma jets based on plasma discharge [9–13] are zero-net-mass-flux flow controls, which require no external flow source. However, both the synthetic and plasma jets, in general, provide low input momentum due to low energy conversion efficiency from electric to fluid power, typically 3% ~ 10% [14–16] for synthetic jets and less than 1% for plasma actuators [17]. The control effectiveness of synthetic and plasma jets is thus limited for high momentum flows.

The flow control methods using fluidic actuators generally can provide high momentum with a high control authority. An example is the widely used circulation control (CC) relying on jet injection and Coanda effect [18–21]. A CC airfoil is effective to enhance lift coefficient. However, CC airfoil is an injection-only flow control method. The PR is path-dependent and a systematic study of PR for CC airfoil is not well documented in published literature.

1.1 Co-Flow Wall Jet

Recently, a fluidic-actuator-based ZNMF Co-Flow Wall Jet (CFWJ) AFC achieves airfoil performance enhancement with ultra-high lift coefficient and high cruise efficiency at low energy expenditure [22–29]. As sketched in Fig. 1, a CFWJ airfoil draws a small amount of mass flow into the airfoil near the trailing edge,

pressurizes and energizes it using a micro-compressor embedded inside the airfoil, and then tangentially injects the same mass flow near the leading edge in the main flow direction.

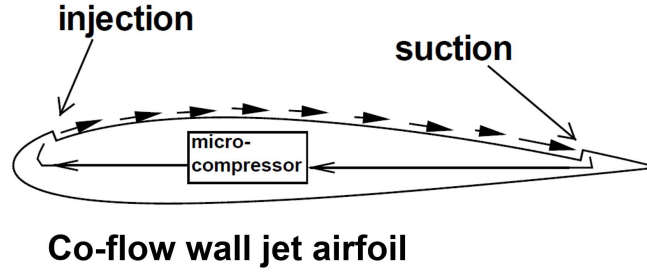


Figure 1: Schematics of the CFWJ airfoil.

The co-flow wall jet has a tangential injection to the wall surface with a streamwise suction downstream. Hence the CFWJ is categorized as a wall jet [30–36]. In this paper, the term “co-flow wall jet” means the same as “co-flow jet (CFJ)”, which is used in other previous publications [22–29, 37–40]. CFWJ and CFJ are interchangeable. CFWJ AFC always comes in pair with an injection and suction to form a closed mass conservation system on the flow control plane. In other words, CFWJ is a ZNMF flow control that combines the features of a wall jet and boundary layer suction. Xu et al [38] investigate the CFWJ separation control mechanism. This paper is focused on the CFWJ energy expenditure.

As shown in Fig. 1, the micro-compressor actuator withdraws the low energy flow downstream caused by total pressure loss, energizes the flow, and injects the high energy flow upstream to energize the boundary layer so that it can overcome severe adverse pressure gradients (APG) [25, 26, 41]. Such energy transfer direction determines that the suction be located downstream and the injection be located upstream. The jet mass flow transportation and energization process through the micro-compressor actuator have the opposite flow direction to the main flow as shown in Fig. 1. The advantage to place the injection upstream is that it requires a relatively low power to eject the flow when the external main flow pressure is low. Similarly, placing the suction downstream with the high external main flow pressure due to APG makes the suction easier. Such an injection and suction distribution is desirable to minimize the energy expenditure. Since this process is local with a short path, it can achieve a high efficiency due to a low total pressure loss, e.g. 1-2% [42] by designing the injection and suction ducts with smooth turning and no flow separation.

Because the CFWJ is a zero-net-mass-flux flow control method, the power required is well defined by the total enthalpy rise from the suction duct outlet (compressor inlet) to the injection duct inlet (compressor outlet) [24]. The total enthalpy rise can be achieved by the embedded micro-compressors or other pumping systems. The power required (PR) at the CFWJ can be expressed as Eq. (1):

$$PR = \dot{m}H_{t2}(\Gamma^{\frac{\gamma-1}{\gamma}} - 1) \quad (1)$$

where \dot{m} is the CFWJ mass flow rate, H_{t2} is the total enthalpy at the suction slot, Γ is the total pressure ratio between the injection and suction, γ is the ratio of specific heat and takes the value of 1.4 for ideal gas.

The power coefficient is defined as:

$$P_c = \frac{PR}{\frac{1}{2}\rho_{ref}U_{ref}^3A_{ref}} \quad (2)$$

where ρ_{ref} and U_{ref} denote the reference density and velocity at the inlet and A_{ref} is the reference area defined as the product of chord length and hump span in this study.

Eq. (1) indicates that the power required by the CFWJ is determined linearly by the mass flow rate and exponentially by the total pressure ratio. This provides a principle to reduce CFWJ power consumption: it is desirable to have a high mass flow rate and low total pressure ratio as the latter increases much faster than the former. This relationship in fact applies to all active flow controls based on fluidic actuators. To be general and focus on the AFC performance, only the power required defined in Eq. (1) is studied in this paper.

The actual total power consumed by CFWJ is

$$P_{total} = PR/\eta \quad (3)$$

where η is the power conversion efficiency (PCE) of the micro-compressor system. The typical micro-compressors used for small general aviation aircraft have a diameter of 5-7 cm, which allows a fairly high power conversion efficiency up to mid 80% [26, 42–44]. If the applications are for large aircraft, the size of the compressors will be also larger and the efficiency could be even higher.

A parameter widely used to describe the jet strength is the jet momentum coefficient C_μ defined below:

$$C_\mu = \frac{\dot{m}U_j}{\frac{1}{2}\rho_{ref}U_{ref}^2A_{ref}} \quad (4)$$

where U_j is the mass-averaged injection velocity. As indicated by Zha et al [27], C_μ only represents a part of the injection thrust. The total reactionary force created by an AFC with fluidic actuator includes the injection thrust and the ram drag created by the suction [27]. Since a flow control with injection-only is not straightforward to calculate the total power by Eq. (1) without the information of the flow path (i.e. Γ), C_μ sometimes was used to represent the added equivalent drag or power coefficient caused by the injection [45], such as $(C_D)_{corrected} = C_D + C_\mu$. However, C_μ does not behave in the same way as the power coefficient quantitatively and qualitatively. The proper way to convert the AFC power required to the added equivalent drag is $(C_D)_{corrected} = C_D + P_c$, where P_c is defined by Eq. (2) [24–26]. A high C_μ could have a lower power consumption than a smaller C_μ if the high C_μ is obtained by a high mass flow rate and low jet velocity, which may result in a significantly lower total pressure ratio [41, 46, 47].

For a typical CFWJ airfoil, the mixing process between the wall jet and main flow starts at the near lowest pressure location of the airfoil behind the leading edge and ends near the highest pressure location upstream of the trailing edge as shown in Fig. 2 (a). The CFWJ covers the area of the adverse pressure gradient (APG). For a CFWJ flight control surface application [29, 48, 49] as shown in Fig. 2 (b), the co-flow wall jet is not extended to the deflected flap area due to structure consideration. The severe APG area of a control surface is mostly in the flap area, which is uncovered by the co-flow wall jet. However, a significant improvement of control surface lift coefficient (C_L) is still achieved with fairly high energy efficiency [29, 48, 49].

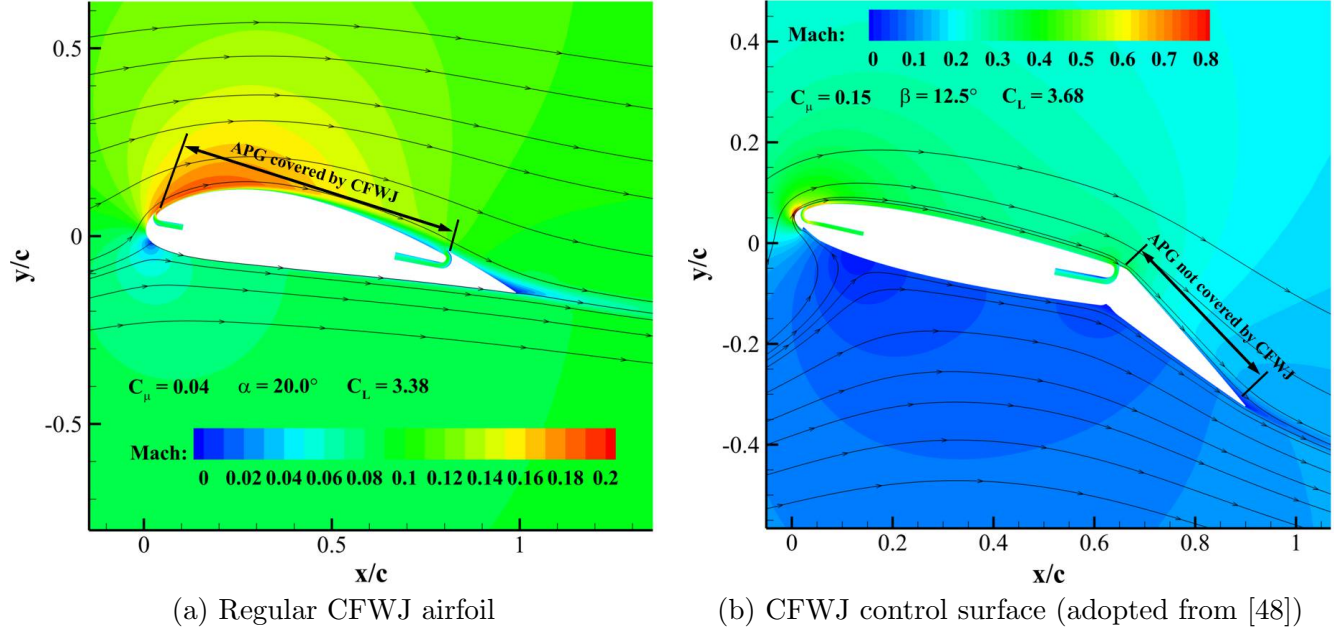


Figure 2: Mach number contours of CFWJ applications.

The above phenomena trigger the following questions: What is the respective role of the injection and suction of CFWJ to minimize its energy expenditure? What is the desirable location to place the CFWJ injection and suction? Is there a coupling effect between the injection and suction? What pressure gradient is more beneficial to CFWJ, adverse pressure gradient or favorable pressure gradient (FPG)? The purpose of this paper is to answer these questions by investigating the energy expenditure of CFWJ using the widely studied NASA hump for separation control. Understanding the energy expenditure is crucial for practical applications.

The NASA hump [50, 51] is often used as a benchmark experiment for various AFC studies, including steady injection [52–54], steady suction [50, 52], unsteady injection [55], synthetic jets [7, 8] and sweeping jets [54, 56, 57]. However, most of these studies focus on control effectiveness, whereas the efficiency or energy expenditure is rarely addressed. Borgmann et al. [54] use energy coefficient (C_E) to measure the power consumption of sweeping jets, which is an injection-only flow control. However, C_E represents the jet kinetic energy, which is similar to the jet momentum coefficient and does not represent the required AFC power determined by the total enthalpy rise. For numerical simulation of ZNMF synthetic jets, the power required to drive the actuator’s reciprocating motion is also difficult to evaluate because it is case-dependent. Benefited from the well-defined power required for CFWJ by Eq. (1), this paper systematically investigates the energy expenditure mechanism of the highly effective CFWJ separation control. The results may apply to general active flow controls with tangential injection and streamwise suction.

2 Numerical Approaches

2.1 Governing Equations

The governing equations for the CFD simulation are the unsteady Reynolds averaged Navier-Stokes equations (URANS) with one equation Spalart-Allmaras turbulence model [58], which are solved in a fully coupled manner using an implicit unfactored Gauss-Seidel line iteration to achieve a high convergence rate [59]. The normalized Navier-Stokes governing equations in generalized coordinates are expressed below:

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{E}}{\partial \xi} + \frac{\partial \mathbf{F}}{\partial \eta} + \frac{\partial \mathbf{G}}{\partial \zeta} = \frac{1}{Re} \left[\frac{\partial \mathbf{R}}{\partial \xi} + \frac{\partial \mathbf{S}}{\partial \eta} + \frac{\partial \mathbf{T}}{\partial \zeta} \right] + \mathbf{S}_\nu \quad (5)$$

where Re is the Reynolds number. The conservative variable vector \mathbf{Q} , inviscid flux \mathbf{E} , viscous flux vector \mathbf{R} are given below, and the rest can be expressed following the symmetric rule.

$$\mathbf{Q} = \frac{1}{J} \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e \\ \rho \hat{\nu} \end{bmatrix}, \mathbf{E} = \frac{1}{J} \begin{bmatrix} \rho U \\ \rho u U + p \xi_x \\ \rho v U + p \xi_y \\ \rho w U + p \xi_z \\ (\rho e + p)U \\ \rho \hat{\nu} U \end{bmatrix}, \mathbf{R} = \frac{1}{J} \begin{bmatrix} 0 \\ \tau_{xi} \xi_i \\ \tau_{yi} \xi_i \\ \tau_{zi} \xi_i \\ (u_j \tau_{ij} - q_i) \xi_i \\ \frac{\rho}{\sigma} (\nu + \hat{\nu}) \frac{\partial \hat{\nu}}{\partial x_i} \xi_i \end{bmatrix}, \mathbf{S}_\nu = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ S_\nu \end{bmatrix}$$

The \mathbf{S}_ν in Eq. (5) is the source term for the S-A model,

$$S_\nu = \frac{1}{J} \left[\frac{1}{Re} \left[-\rho (c_{w1} f_w - \frac{c_{b1}}{\kappa^2} f_{t2}) \left(\frac{\hat{\nu}}{d} \right)^2 \right] + \frac{1}{Re} \left[\frac{\rho}{\sigma} c_{b2} (\nabla \hat{\nu})^2 - \frac{1}{\sigma} (\nu + \hat{\nu}) \nabla \hat{\nu} \bullet \nabla \rho \right] + Re \left[\rho f_{t1} (\Delta q)^2 \right] + \rho c_{b1} (1 - f_{t2}) \tilde{S} \hat{\nu} \right] \quad (6)$$

Other auxiliary relations and coefficients for the S-A turbulence model can be found in [28, 58].

2.2 Navier-Stokes Equations Solver

The in-house high order CFD code Flow-Acoustics-Structure Interaction Package (FASIP) is used to solve the 2D URANS equations. A 3rd order WENO scheme for the inviscid flux [59–64] and 4th order central differencing for the viscous terms [60, 64] are employed to discretize the Navier-Stokes equations. The low diffusion E-CUSP scheme suggested by Zha et al [61] based on the Zha-Bilgen flux vector splitting [65] is utilized with the WENO scheme to evaluate the inviscid fluxes. All the simulations in this study are conducted as unsteady time-accurate simulations. The second order time accurate implicit method with pseudo time and Gauss-Seidel line relaxation is used to achieve a fast convergence rate [59, 66]. Parallel computing is implemented to save wall clock simulation time [66, 67]. The numerical results are time-averaged results after the flows and all the aerodynamic forces become dynamically stable. The FASIP code is intensively validated for CFWJ simulations [22–25, 27, 29, 67–72]. Particularly, the predicted power coefficient P_c of CFWJ agrees very well with the experiment [24], which provides a solid support for the energy prediction in the present study. Rumsey [51] compared different turbulent models for the NASA

hump simulation. The S-A one equation model performs reasonably well to predict the surface pressure distribution, separation onset and separation bubble length. It behaves as well as other RANS models such as SST model.

3 The NASA Hump

The NASA hump is widely used as a benchmark case to validate numerical algorithms and turbulence modeling [51] for flow control. The baseline hump configuration with no flow control is designed to have a converging section followed by a rapid area expansion downstream of the throat as shown in Fig. 3, which creates a severe diffusion and large flow separation.

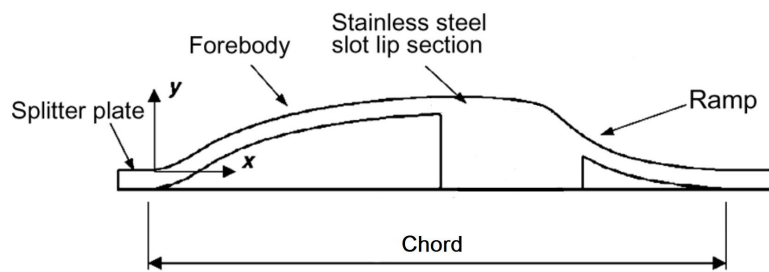


Figure 3: Geometry of the hump upper surface [50].

To provide the pressure gradient distribution information of the NASA hump when the separation is removed after applying flow control, an inviscid simulation of the hump that has no flow separation is conducted. Fig. 4 presents the Mach number contours on the left, and the pressure coefficient C_p and pressure gradient $\partial p/\partial x$ distributions on the right with the mesh refinement results showing the mesh independent solutions. The region of the favorable pressure gradient (FPG) is from 30% c to 65% c and the adverse pressure gradient (APG) is from 65% c to 80% c .

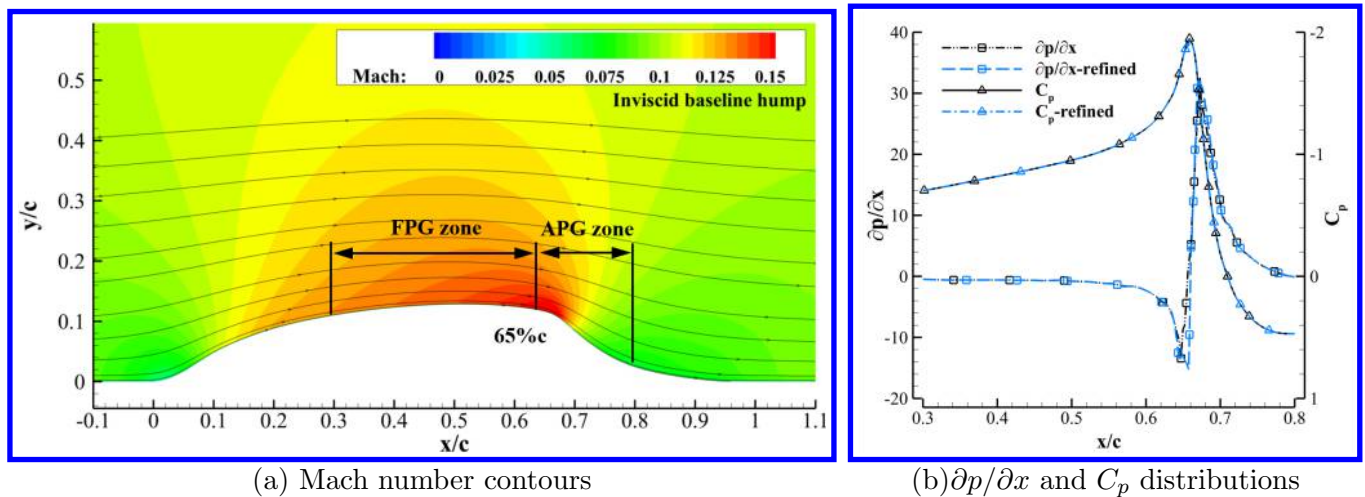


Figure 4: Results of the baseline hump with inviscid flow simulation.

The viscous flow of the baseline geometry is simulated with an initial mesh of $408 \times 108 = 44,064$ cells. The mesh topology and boundary condition set-up following reference [51] are discussed in Ref. [38]. Fig. 5 (a) is the Mach number contours displaying a large flow separation downstream of the baseline hump. Fig. 5 (b) gives the corresponding C_p and $\partial p/\partial x$ distributions of the viscous results. The present numerical simulation is in a very good agreement with the experiment for the C_p distribution and separation onset location at $x/C=66.3\%$. The measured onset point in the experiment is at $66.5\%c$. The predicted separation onset location is determined using the skin friction distribution presented in Ref. [38]. The mesh refinement study is also conducted by doubling the grid points in both directions. The reattachment point is slightly over-predicted as reported by other research groups using RANS models [51, 73]. The mesh refinement study indicates that the solutions are converged based on the initial mesh size. The unsteady simulation uses a constant non-dimensional characteristic time step $\Delta \bar{t} = 5 \times 10^{-3}$ with the maximum L2-norm residual typically reduced by 2 orders of magnitude within less than 40 pseudo time iterations per physical time step. Details of the residual convergence histories are presented in Ref. [38].

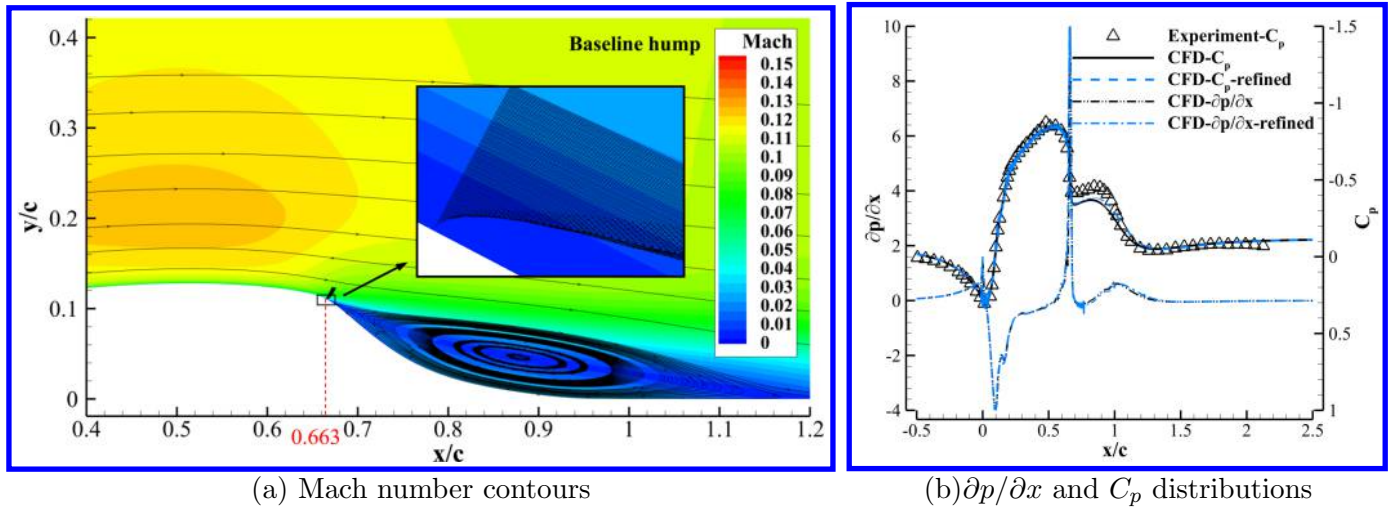


Figure 5: Results of the viscous baseline hump.

Two experimental cases with active flow control are used to validate the present numerical simulation [38], steady injection [54] and steady suction cases [51]. The steady injection case validates the CFWJ injection simulation and the steady suction case validates the CFWJ suction prediction. The numerical validation results agree well with the experiments for both cases in terms of flow field structures (e.g. separation mitigation) and C_p distribution. To avoid redundancy and save space, the details of those validations are not presented in this paper and can be found in Ref. [38].

4 Co-Flow Wall Jet Hump

4.1 CFWJ Hump Geometry

The CFWJ hump configurations are created as described in Ref. [38] and are illustrated in Fig. 6. The surface between the injection and suction slots is slightly moved downwards by $0.1\%c$ to facilitate the tangential injection of the co-flow wall jet.

Similar to a CFWJ airfoil, Fig. 6 (middle plot) shows that a small amount of mass flow is drawn into the hump downstream, pressurized and energized by a micro-compressor pumping system inside the hump, and ejected through the upstream injection slot tangential to the main flow. In the present simulation, the micro-compressor actuator is simulated by applying total pressure inlet boundary condition (BC) at injection slot and static pressure outlet BC at suction slot as shown in the bottom plot of Fig. 6. The calculation has two layers of iterations: 1) matching the specified C_{μ} ; 2) matching the suction mass flow rate to the injection one for ZNMF. This method is validated in the previous work [22–25, 27–29, 48, 68–72].

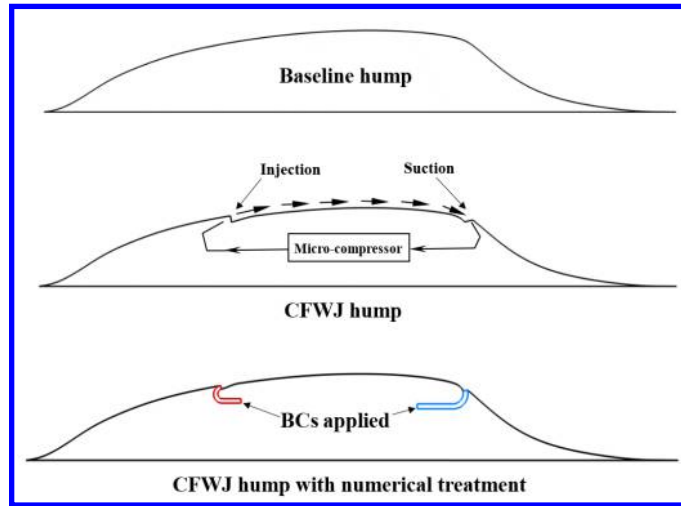


Figure 6: Sketch of the CFWJ hump.

As indicated in Ref. [38], it is effective to suppress flow separation by applying flow control near the separation onset point in adverse pressure gradients. Since CFWJ has both the injection and suction simultaneously, the present research will have the cases with either the injection or suction placed at the separation onset point to study their individual effect. CFWJ is devised to place the suction downstream and injection upstream. When the suction slot is placed near the separation onset point, the injection has to be located upstream with the favorable pressure gradient. Based on the regions with different pressure gradients indicated in Fig. 4, the present study hence has two categories of pressure gradients that the injection slot is located: 1) adverse pressure gradient when the injection is placed near the separation onset point; 2) favorable pressure gradient when the suction is placed near the separation onset point.

Even though this paper primarily studies the injection and suction location effect, the slot size of injection and suction also plays a very important role in energy expenditure. McGahan [74] observed that a larger size of wall jet injection slot reduces more energy consumption. But there is an optimized slot size. Our studies agree with McGahan’s observation [24, 28, 41, 47]. The reason is that a larger injection slot size will reduce the jet velocity and total pressure loss of the CFWJ, which requires a lower total pressure ratio of the micro-compressors. As explained by Eq. (1), the CFWJ power required is determined exponentially by the total pressure ratio. However, the injection slot size should not be too large to lose the required jet momentum. The slot size for this study is already optimized based on our previous experience. Thus the focus of the trade study is on the injection and suction slot location to investigate their effect on energy expenditure in different pressure gradients.

4.2 Injection in Adverse Pressure Gradients

The slot sizes of $0.5\%c$ for injection and $0.7\%c$ for suction are adopted for all the cases. The C_μ of 0.85% is used for all the cases because this is the minimum value that makes one of these cases fully attached. The purpose is to compare the behavior of different CFWJ location effects with the same injection strength.

4.2.1 Injection Location Trade Study

The CFWJ configurations with injection located at $65\%c$, $67.5\%c$, and $70\%c$ shown as Case 1, 2, and 3 in Fig. 7 are first studied. The suction slot is fixed at $90\%c$ location for all these cases. Among the three injection locations, the $67.5\%c$ is the closest to the separation onset location of $66.5\%c$. The injection slot at $65\%c$ and $70\%c$ are not drawn in Fig. 7 but are indicated by the arrows for the clarity of the plot.

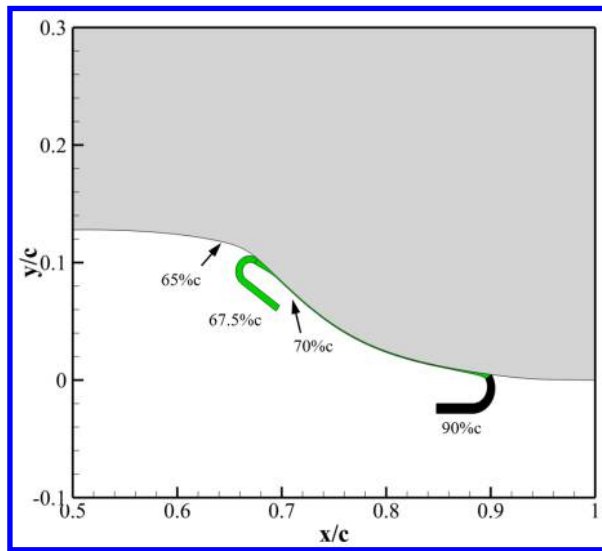


Figure 7: Injection location trade study (Case 1, 2, 3).

Fig. 8 shows the Mach number contours of the injection location trade study with C_μ of 0.85% . Flow separation is observed for Case 1 downstream of the injection located at $65\%c$. As the injection location is moved to $67.5\%c$ for Case 2, $1\%c$ downstream of the separation onset point, flow is fully attached. Placing the injection at the $65\%c$ is still able to remove the flow separation, but needs to increase the C_μ to 1.7% , 100% higher than 0.85% . Case 3 with the injection located at $70\%c$ always has a small separation upstream of the injection slot, which is not able to be removed even with a very large C_μ .

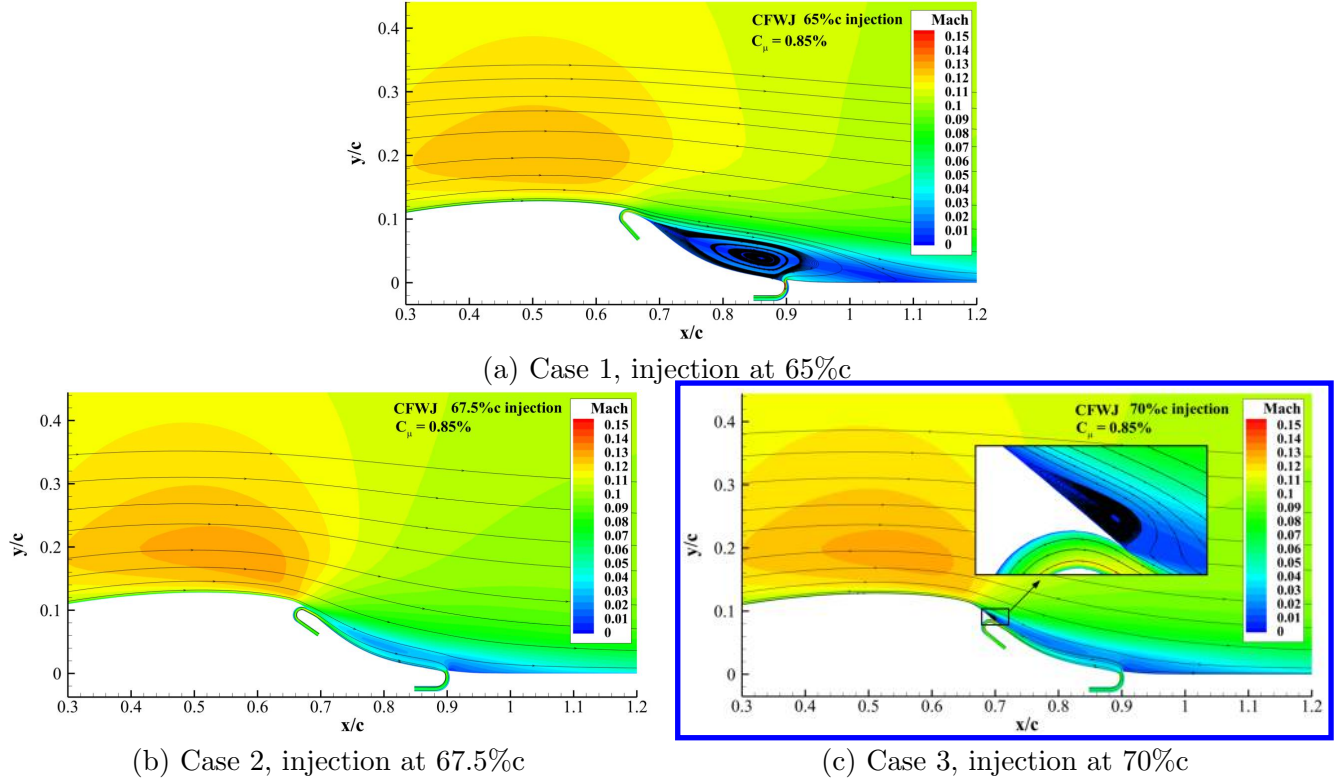


Figure 8: Injection location trade study with the suction fixed at 90%c position.

Table 1 compares the power consumption of the two APG CFWJ injection cases that are able to remove the separation with their respective minimum required C_{μ} . Case 3 with the injection at 70%c is not included because it is unable to remove the small flow separation upstream of the injection slot. The \bar{m} is the normalized CFWJ mass flow rate, defined as $\dot{m}/(\rho_{ref}U_{ref}A_{ref})$ and Γ is the total pressure ratio between CFWJ injection and suction. The Re_h is the Reynolds number based on the injection slot height. The CFWJ power coefficient P_c of Case 2 is 50% smaller than that of Case 1. It indicates that placing the injection slightly downstream of the separation onset point is much more efficient than placing it upstream. The 50% power reduction is attributed to the 29% smaller mass flow rate and the rest is attributed to the 0.15% smaller total pressure ratio, which indicates that a very small reduction of the total pressure ratio such as 0.15% can reduce the total pressure ratio term in Eq. (1) by 30% due to the exponential effect.

Table 1: Power consumption of Case 1 and 2 with APG injection

Cases	Inj	Suc	C_{μ}	Re_h	U_{jet}/U_{ref}	P_c	\bar{m}	Γ	Flow
CFWJ 1	65%c	90%c	1.70%	6236	1.31	0.0060	0.65%	1.0060	Fully attached
CFWJ 2	67.5%c	90%c	0.85%	4379	0.92	0.0030	0.46%	1.0045	Fully attached

4.2.2 Suction Location Trade Study

Since the injection location at 67.5%c is the most efficient position for the injection trade study, the injection is thus fixed at 67.5%c and has the suction varied at four positions: 70%c, 75%c, 80%c, and 85%c

(Case 4, 5, 6, and 7) as shown in Fig. 9. The suction at 70% c position is not drawn but is indicated by an arrow for the plot clarity.

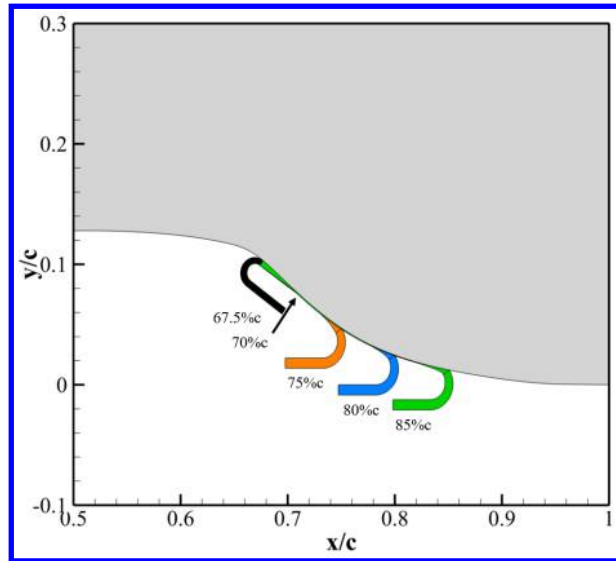


Figure 9: Suction location trade study (Case 4, 5, 6, 7).

Fig. 10 shows the Mach number contours of suction location trade study Case 4, 5, 6, and 7. All these cases have the same C_μ of 0.85%. For Case 4 with the suction at 70% c (Fig. 10 (a)), the closest to the injection slot, large flow separation occurs. With the suction slot moved downstream to 75% c and 80% c (Fig. 10 (b) and (c), Case 5 and 6), the flow separation is mostly removed with a tiny separation bubble that can only be observed when it is zoomed in. When the suction is located at 85% c (Fig. 10 (d), Case 7), the flow separation is completely removed, the same as Case 2 with the suction located at 90% c .

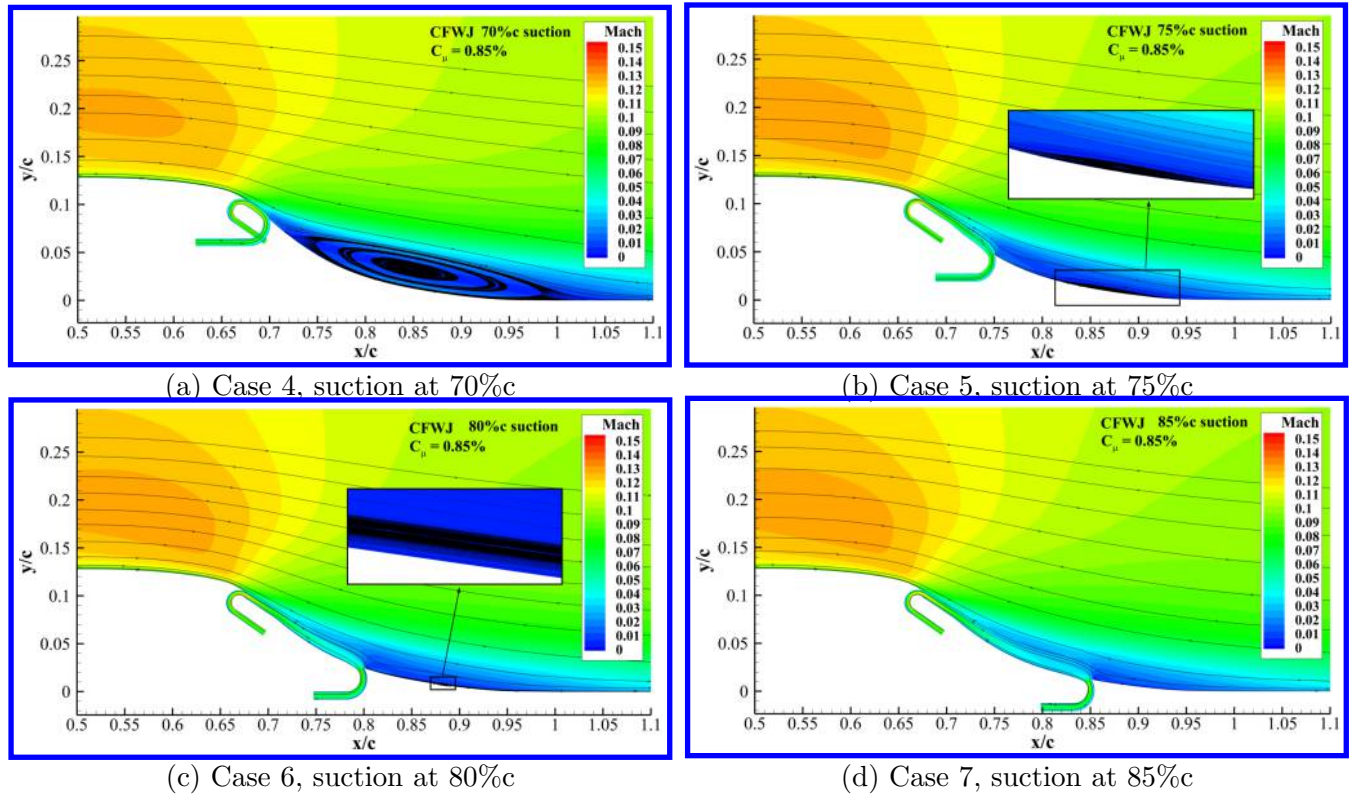


Figure 10: Suction location trade study with the injection fixed at 67.5% c position.

Fig. 11 compares the normalized total pressure contours of Case 5 with Case 7 to show the energy transfer difference between the flow slightly separated and fully attached. Note all these suction trade study cases have the same injection strength, injection total pressure, injection location, and C_{μ} . For Case 5 shown in Fig. 11 (a), the jet still has high energy when it is withdrawn into the suction slot because it does not have enough length to mix with the main flow and transfer the energy. The boundary layer is thus not energized enough to overcome the adverse pressure gradient and is slightly separated as shown in Fig. 10 (b). For Case 7 as shown in Fig. 11 (b), a longer distance between the suction and injection slot makes the jet mixing more thoroughly. The lower total pressure at the suction slot is due to more energy transferred from the jet to the main flow boundary layer by mixing. In other words, a sufficiently long distance for mixing between the injection and suction of CFWJ is beneficial to suppress the flow separation.

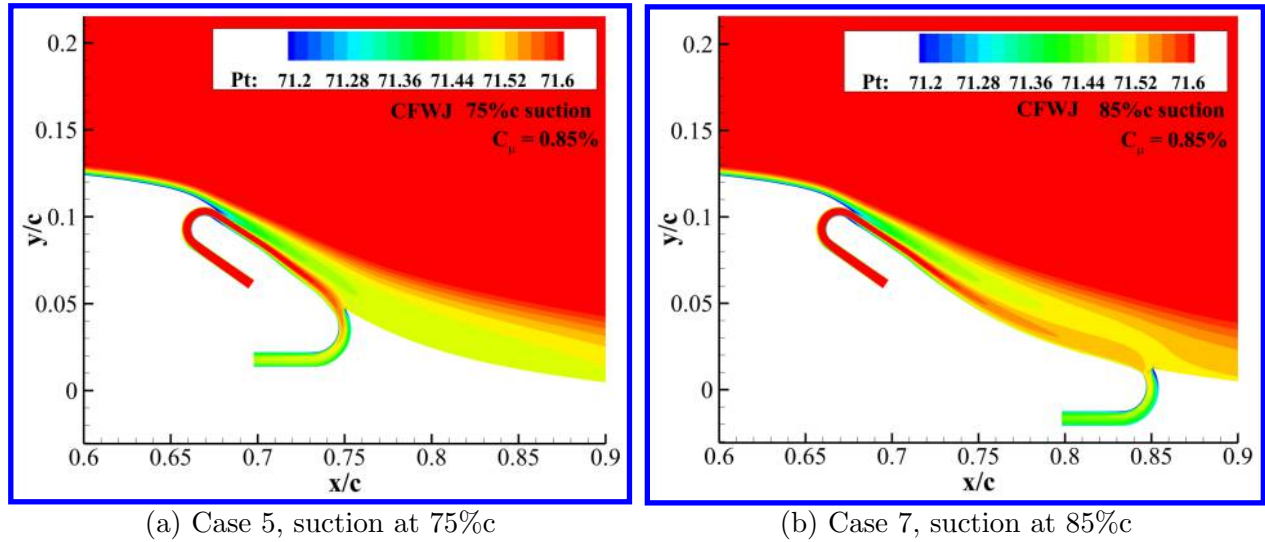


Figure 11: Total pressure comparison with the injection fixed at 67.5%c.

The flow of Case 4, 5, and 6 can be fully attached when their corresponding C_μ is increased. Table 2 compares the power consumption for Case 4, 5, 6, and 7 with their minimum C_μ that attaches the flow. With the suction slot moved downstream from 70%c to 85%c, the C_μ that is able to attach the flow is substantially decreased by 72% from C_μ of 3% to 0.85%. The CFWJ power is dropped by a similar amount of 73%. For Case 7 and Case 2, the only difference is that the suction is at 85%c for the former and at 90%c for the latter (Table 2). They have the same CFWJ power coefficient P_c of 0.0030. In this case, the injection close to the separation onset point has the dominant effect. The suction has a weak coupling effect to suppress the flow separation, but mainly serves as the flow source to achieve ZNMF.

Table 2: Power consumption of various suction location cases with APG injection

Cases	Inj	Suc	C_μ	Re_h	U_{jet}/U_{ref}	P_c	\bar{m}	Γ	Flow
CFWJ 2	67.5%c	90%c	0.85%	4379	0.92	0.0030	0.46%	1.0045	Fully attached
CFWJ 4	67.5%c	70%c	3.00%	8216	1.73	0.0110	0.86%	1.0090	Fully attached
CFWJ 5	67.5%c	75%c	1.00%	4760	1.00	0.0034	0.50%	1.0048	Fully attached
CFWJ 6	67.5%c	80%c	0.90%	4512	0.95	0.0032	0.47%	1.0048	Fully attached
CFWJ 7	67.5%c	85%c	0.85%	4379	0.92	0.0030	0.46%	1.0046	Fully attached

Three conclusions may be drawn from these trade study results with CFWJ APG injection: 1) The injection of CFWJ placed slightly downstream of the separation onset point is most effective and efficient to remove flow separation in adverse pressure gradients. 2) The distance between the injection and suction slot needs to be long enough so that the CFWJ can be fully mixed with the main flow to transfer the energy. 3) When the injection is located at the desirable position slightly downstream of the separation onset point and the mixing distance is sufficiently long, the injection effect is dominant and the suction has a weak coupling effect, but primarily serves to withdraw the mass flow as the source for the injection.

4.3 Injection in Favorable Pressure Gradients

When the CFWJ suction is placed near the separation onset location, the CFWJ injection will be placed upstream in the region of the favorable pressure gradient. This section is to conduct the trade study with the injection located in the FPG region. Similar to the last section, the trade study also consists of two parts: injection and suction location studies. The slot size of $0.5\%c$ for injection, $0.7\%c$ for suction and C_μ of 0.77% are used for all the cases.

4.3.1 Suction Location Trade Study

Fig. 12 shows the various suction locations for the trade studies: Case 8, 9, 10, 11, 12, and 13, which have the injection location fixed at the FPG position of $50\%c$ and the suction is varied in the region of APG at $67.5\%c$, $70\%c$, $72\%c$, $75\%c$, $80\%c$, and $90\%c$.

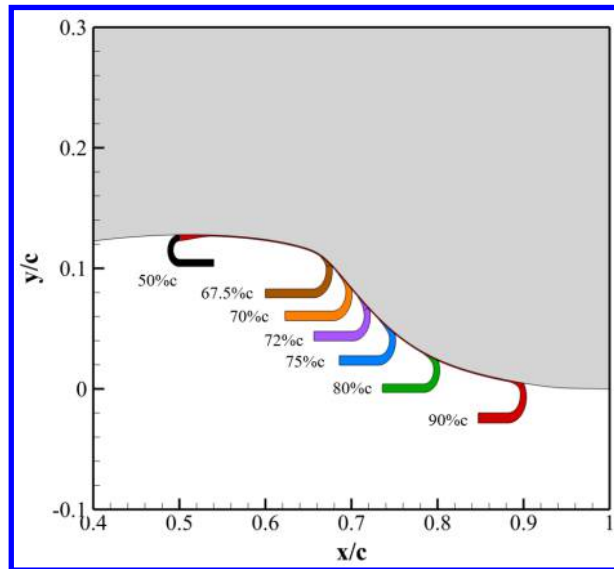
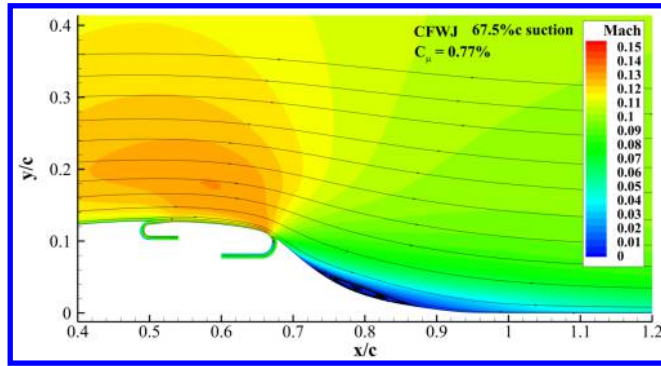
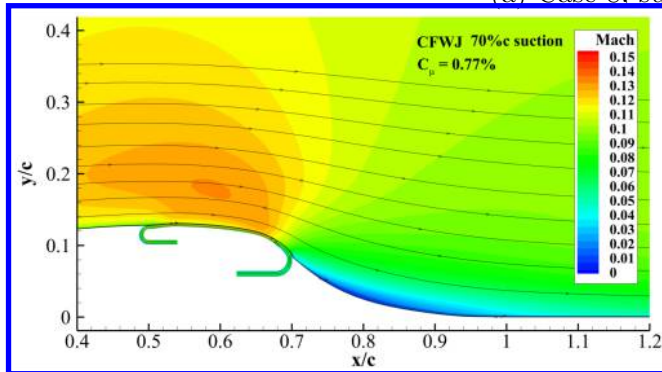
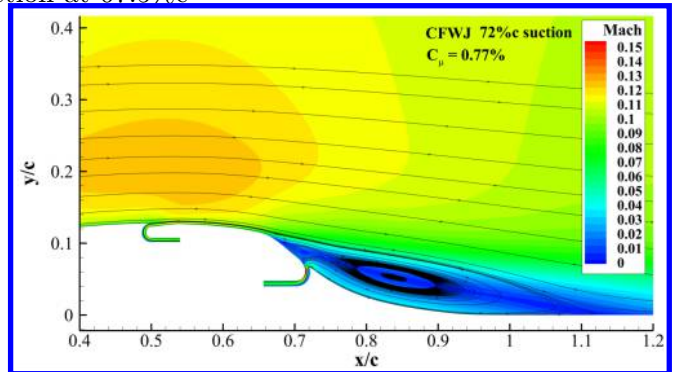


Figure 12: Suction location trade study (Case 8-13).

For Case 8 with the suction located at $67.5\%c$, the flow is slightly separated as shown in Fig. 13 (a). With the suction moved to $70\%c$ location (Fig. 13 (b)) for Case 9, the flow is fully attached. If the suction location is moved $2\%c$ further downstream to $72\%c$ for Case 10, a large flow separation occurs at $0.65 < X/C < 1.15$. Further moving the suction slot downstream to $75\%c$, $80\%c$ and $85\%c$ results in a large flow separation as for the $72\%c$ location and are not shown. These results support the analysis in Ref. [38] that applying a flow control such as a suction close to the separation onset point is effective to suppress the flow separation. However, the criterion to determine the optimal distance placing a flow control actuation downstream of the separation onset point remains to be studied.

(a) Case 8, suction at 67.5% c (b) Case 9, suction at 70% c (c) Case 10, suction at 72% c Figure 13: Suction location trade study with the injection fixed at 50% c .

Referring to Fig. 13, Fig. 14 compares the velocity profiles for Case 8 (mildly separated) and Case 9 (fully attached) with the same C_{μ} of 0.77% at a location immediately downstream of the suction slot (Station 1) and a common location at 72% c (Station 2). Case 8 has a much stronger boundary layer profile immediately downstream of the suction slot than Case 9. However, the momentum is quickly dissipated and the boundary layer is weaker than that of Case 9 at the location of 72% c . This is because the suction is not far enough from the injection to provide sufficient distance to energize the boundary layer via mixing. The jet is withdrawn into the suction slot too early and is still in a high-energy state. The suction location at 70% c of Case 9 provides a longer distance to make full use of the kinetic energy from the injection. Table 3 quantifies the displacement thickness δ^* and shape factor H of the boundary layer profile at Station 1 and 2 for Case 8 and 9 respectively. At Station 1, Case 8 has a more healthy boundary layer profile than Case 9 in terms of smaller displacement thickness (δ^*) and H factor. However, as the flow travels downstream to Station 2, the H factor and δ^* of Case 8 increase dramatically and exceed those of Case 9.

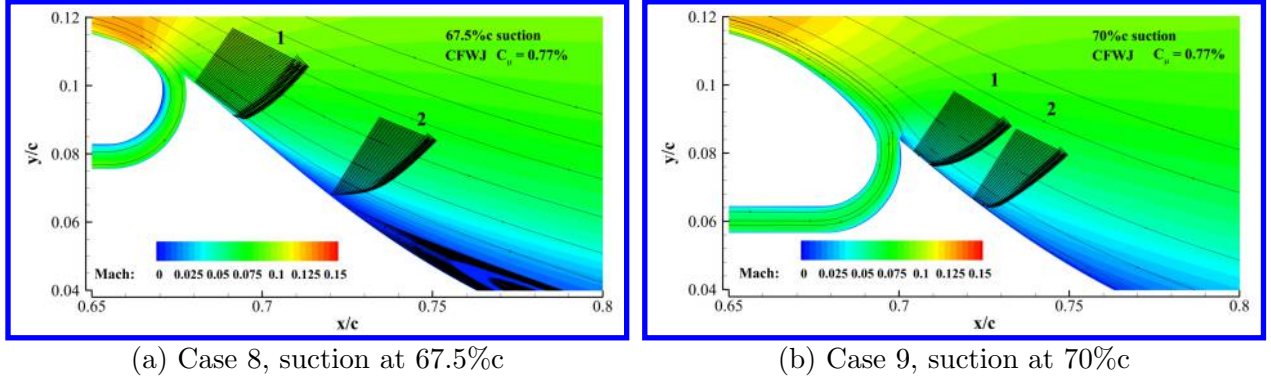


Figure 14: Mach number contours and velocity profiles.

Table 3: Details of velocity profiles of Case 8 and Case 9

Case	C_μ	Station 1		Station 2	
		δ^*	H	δ^*	H
CFWJ 8	0.77%	2.6×10^{-3}	1.18	1.35×10^{-2}	1.96
CFWJ 9	0.77%	8.84×10^{-3}	1.58	1.21×10^{-2}	1.66

For Case 8 and 10 with flow separation shown in Fig. 13, the flow can be attached by increasing C_μ . Table 4 lists the results with their corresponding minimum C_μ that makes the flows attached with the injection fixed at 50%c. A significant difference of the suction location trade study from the injection trade study in Section 4.2 is that all cases with different suction locations including the one at deep separation region of 90%c can attach the flow, but at a high energy cost as shown in Table 4. The most effective injection location at APG from the trade study in Section 4.2 is downstream of the baseline separation onset point at 67.5%c. However as shown in Table 4 for Case 8, placing the suction at this location has the C_μ and P_c increased by 69% and 109% respectively compared with placing it at 70%c. In other words, placing the injection or suction slot slightly downstream of the separation onset point is the most effective for separation removal. But the desirable suction location is a little downstream of the optimal injection location. Note that the optimal location herein refers to the most effective location based on the current hump configurations and the limited trade study cases. The quantitative results should not be generalized and are not conclusive.

Table 4 indicates that Case 9 with the suction location at 70%c is the optimum among the cases of the suction location trade study. Placing the suction upstream or downstream of this location consumes more energy.

Table 4: Power consumption of various suction location cases with FPG injection

Cases	Inj	Suc	C_μ	Re_h	U_{jet}/U_{ref}	P_c	\bar{m}	Γ	Flow
CFWJ 8	50%c	67.5%c	1.30%	5474	1.15	0.0067	0.56%	1.008	Fully attached
CFWJ 9	50%c	70%c	0.77%	4189	0.88	0.0032	0.44%	1.005	Fully attached
CFWJ 10	50%c	72%c	1.40%	5664	1.19	0.0080	0.59%	1.009	Fully attached
CFWJ 11	50%c	75%c	1.75%	6283	1.32	0.0110	0.66%	1.012	Fully attached
CFWJ 12	50%c	80%c	2.10%	6902	1.45	0.0160	0.72%	1.015	Fully attached
CFWJ 13	50%c	90%c	2.30%	7187	1.51	0.0170	0.75%	1.016	Fully attached

4.3.2 Injection Location Trade Study

For the injection location study as illustrated in Fig. 15, three locations of 30%c, 60%c, and 65%c (Case 14, 15, and 16) in the FPG area are simulated with suction fixed at 70%c, which is the optimal suction location from Case 9. The injection at 65%c is not plotted in Fig. 15 but is indicated by an arrow for clarity.

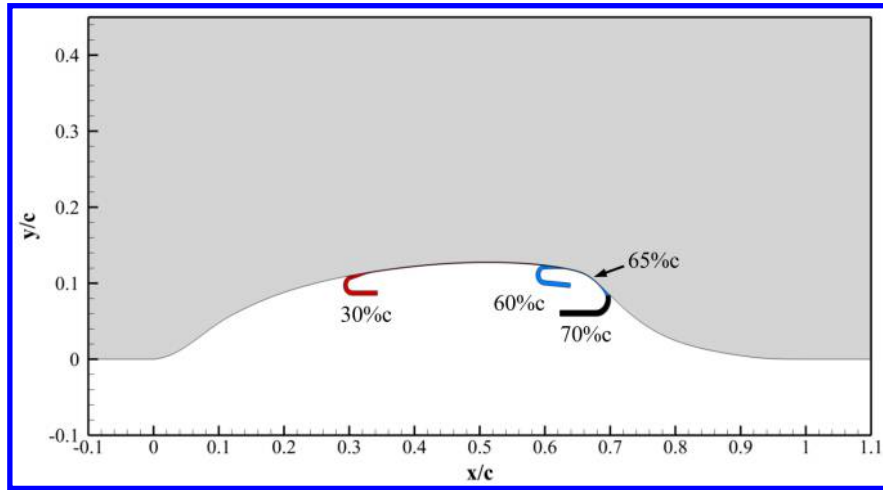


Figure 15: Injection location trade study (Case 14, 15, and 16).

Fig. 16 shows the Mach number contours of the CFWJ hump at injection locations of 30%c, 60%c, and 65%c with $C_\mu=0.77\%$. Case 14 with injection at 30%c is able to completely remove the separation. Case 15 and 16 with injection located more downstream closer to the suction slot have flow separation occurred due to the insufficient mixing distance, which is the same reason as for Case 4, 5, and 6.

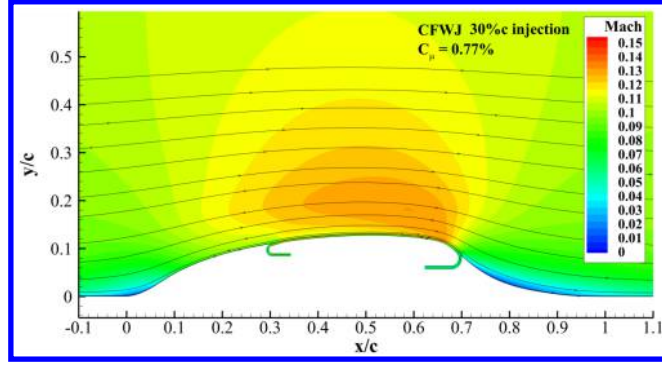
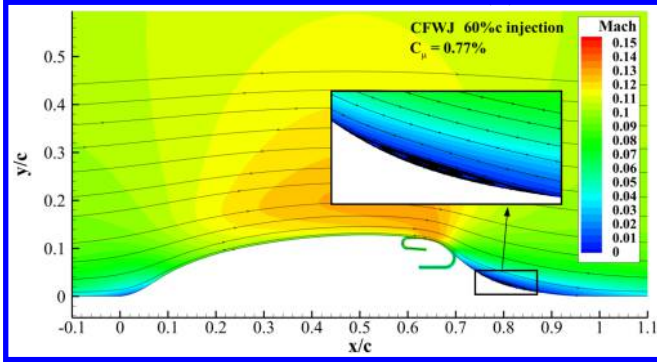
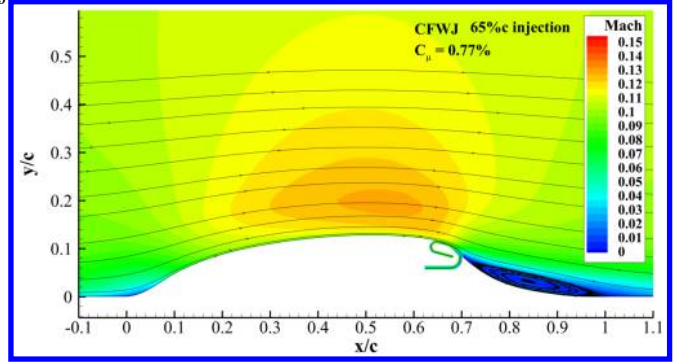
(a) Case 14, injection at 30% c (b) Case 15, injection at 60% c (c) Case 16, injection at 65% c Figure 16: Injection location trade study with the suction fixed at 70% c .

Table 5 compares the energy consumption of the trade study of FPG injection cases. All cases in Table 5 use the minimum C_μ that makes the respective flow fully attached. Comparing Case 14 and 15, Case 15 has the C_μ and mass flow rate increased by 47% and 23% respectively, but the power coefficient is reduced by 2.6%. It is because the total pressure ratio of Case 15 is 0.1% lower than that of Case 14 as explained by Eq. (1). However, both Case 14 and 15 have a higher P_c than Case 9 that has the injection location at 50% c with a P_c of 0.0032. Case 9 has the injection located between that of Case 14 and 15. It combines the advantages of the low mass flow rate of Case 14 and the low total pressure ratio of Case 15. It thus has the minimum CFWJ power coefficient. Case 9 is therefore regarded as the optimum of the FPG injection CFWJ.

Table 5: Power consumption of various injection location cases with FPG injection

Cases	Inj	Suc	C_μ	Re_h	U_{jet}/U_{ref}	P_c	\bar{m}	Γ	Flow
CFWJ 9	50% c	70% c	0.77%	4189	0.88	0.0032	0.44%	1.005	Fully attached
CFWJ 14	30% c	70% c	0.75%	4142	0.87	0.0039	0.43%	1.006	Fully attached
CFWJ 15	60% c	70% c	1.10%	5046	1.06	0.0038	0.53%	1.005	Fully attached
CFWJ 16	65% c	70% c	1.90%	6616	1.39	0.0059	0.59%	1.006	Fully attached

4.4 Comparison of the Optimal Cases with Injection in APG and FPG

Fig. 17 compares the minimum power coefficients (P_c) for all the trade study cases with the flow fully attached. Case 3 that is not able to fully attach flow is omitted in the plot. The solid square symbol represents the injection location studies in APG (Case 1 and 2, Table 1), the solid triangle symbol represents the suction location studies with the injection in APG (Case 4-7, Table 2), the open diamond symbol represents the suction location studies with the injection in FPG (Case 8-13, Table 4) and the open circle symbol represents the injection location studies in FPG (Case 14-16, Table 5). Fig. 17 indicates that the minimum P_c is achieved by Case 2 and Case 9 that have their injection or suction located near the separation onset location and slightly downstream. It also shows that the optimal suction location (Case 9) is a little more downstream than that of the optimal injection location (Case 2).

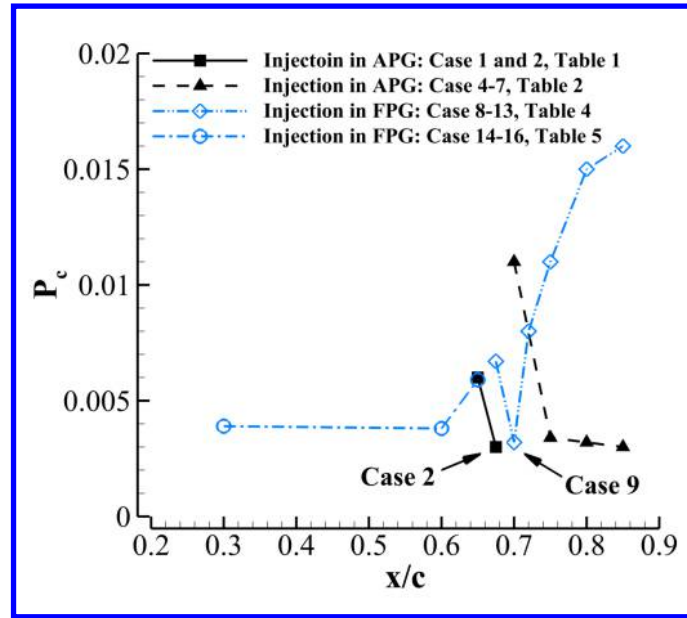


Figure 17: Power coefficient versus locations.

Since Case 2 and 9 are the two most efficient cases, investigation of the flow structure difference between them is useful to understand the effect of injection in APG (Case 2) and injection in FPG (Case 9). Fig. 18 (b) shows the C_p distributions of the computed baseline NASA hump, Case 2 and Case 9 compared with that of the baseline with no flow control. The pressure reaches the lowest value at the negative C_p peak location and immediately experiences a sharp rise due to the rapid area expansion with the attached flow, which forms the pressure spikes at about 66% c . The sharp peak spike is also observed in the inviscid results as shown in Fig. 4 (b) and also measured in the experiment when the flows are attached [38, 51, 54]. The C_p distribution of the two CFWJ cases are very similar, but Case 9 has a higher suction peak and lower pressure valley than Case 2, which indicates that Case 9 experiences a greater acceleration and deceleration (diffusion) than that of Case 2. Such a larger velocity variation of Case 9 creates more energy loss and needs higher CFWJ power as indicated in Table 6 below.

The C_p distribution has a pressure glitch caused by the numerical resolution at the hump leading edge of 0% c . This is due to the sharp change of the pressure caused by an abrupt geometry slope at the hump leading edge, which is not fully resolved by the 3rd order WENO scheme. However, the minor glitch does not change the nature of the flow field and energy analysis. The same numerical glitch is also reported by

other researchers, including Rumsey [51], Koklu [55], and Uzun and Malik [75]. The rest of the pressure spikes are caused by the injection slots and suction slots for three reasons: 1) The pressure probes are along the wall surface without going into the ducts and skip the injection and suction slot opening. The pressure is not uniform across the injection and suction slot opening and appears as spikes when the pressure probe jumps from the injection slot lip to the hump surface and vice versa at the suction slot. 2) The injection and suction slot opening surface is a part of a control volume boundary for force integral and power required computation [24, 27]. The slot opening surface is oriented to be approximately normal to the flow. The coordinates across the slot opening surface hence have a very small discontinuity as shown in Fig. 18. 3) The sharp lip of the suction slot creates a very small acceleration zone followed by a stagnation zone, around which the flow generates a rapid pressure decrease and increase shown as spikes.

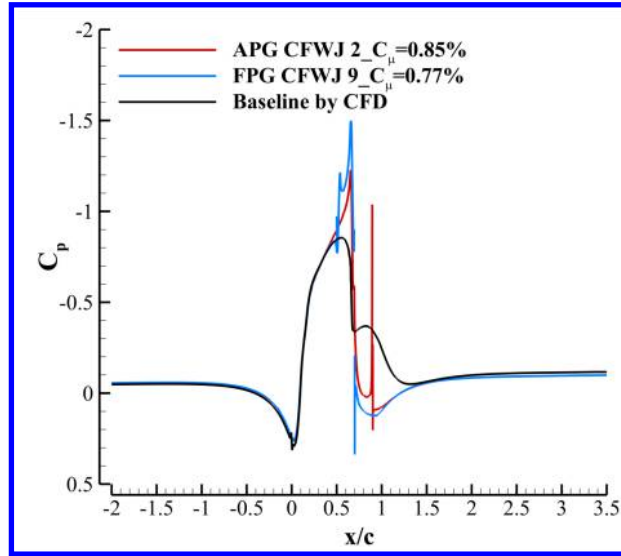


Figure 18: C_p distributions for the baseline and the optimum CFWJ cases.

For comparison convenience, the power coefficient results of Case 2 and 9 are extracted from Table 1 and 4 and are presented here in Table 6.

Table 6: Performance Comparison of APG injection Case 2 and FPG injection Case 9

Cases	Inj	Suc	C_μ	Re_h	U_{jet}/U_{ref}	P_c	\bar{m}	Γ	Flow
CFWJ 2	67.5% c	90% c	0.85%	4379	0.92	0.0030	0.46%	1.0045	Fully attached
CFWJ 9	50% c	70% c	0.77%	4189	0.88	0.0032	0.44%	1.0050	Fully attached

Comparing the P_c between the optimum APG injection and the optimum FPG injection, the optimum FPG injection Case 9 requires a P_c about 6.7% higher than the optimum APG injection Case 2 but has a 10% lower C_μ . This is because Case 9 has a higher total pressure loss that needs a greater total pressure ratio Γ even though its mass flow rate is lower. This also indicates that C_μ should not be used to represent the power consumption of AFC since a high C_μ does not mean a high power consumption. Furthermore, the quantitative values of C_μ and P_c are dramatically different. Using C_μ instead of P_c to represent the added equivalent drag would create misleading results.

Fig. 19 compares the contours of entropy increase (from the inlet) and velocity magnitude (normalized

by the inlet velocity) for Case 2 and Case 9. As shown in Fig. 19 (a) and (b), the APG injection Case 2 has a significantly lower entropy increase than that of the FPG injection Case 9 within the CFWJ region due to the lower main flow velocity and shear stress work [76,77]. For the FPG injection Case 9, the injection jet has strong acceleration due to the favorable pressure gradient as shown in Fig. 19 (d), resulting in a high velocity in the CFWJ region and high energy loss. For Case 2, the velocity in the CFWJ region is substantially lower than that of Case 9 due to the severe adverse pressure gradient and flow diffusion as shown in Fig. 19 (c) and (d). The loss comparison of Fig. 19 is consistent with the co-flow wall jet separation control analysis conducted in Ref. [38], which indicates that it is more efficient and effective to apply the co-flow wall jet in the adverse pressure gradient region starting from the near separation onset location.

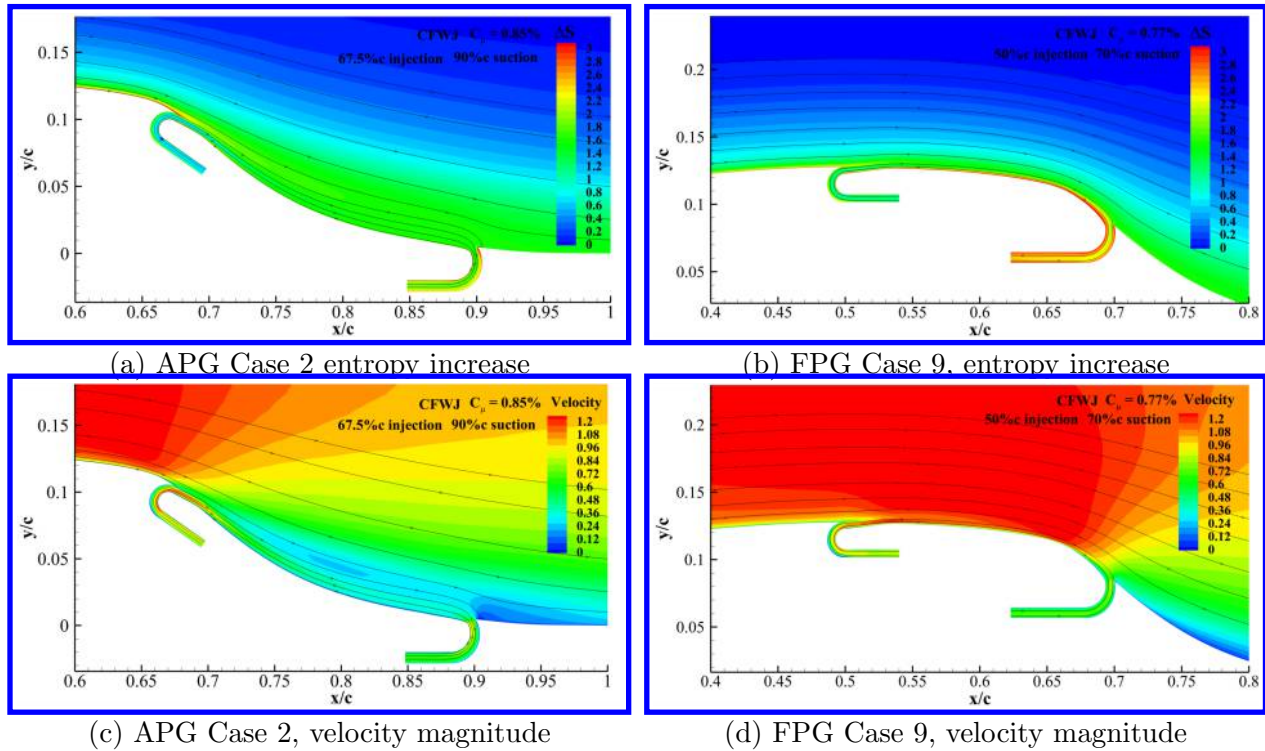


Figure 19: Comparison between the optimum APG and FPG cases.

Fig. 20 compares the spanwise vorticity contours for Case 2 and 9. For Case 2 with the CFWJ completely immersed in the APG region, the three counter-rotating layers of vorticity are observed clearly downstream of the injection slot (Fig. 20 (a)), a layer of clockwise vorticity due to the wall boundary layer in blue, a layer of counter-clockwise vorticity due to the wall jet in red, a zero-vorticity layer indicating a transition of vorticity direction in green, and a layer of clockwise vorticity due to the main flow boundary layer mixing with the wall jet in blue. With the CFWJ flowing downstream, it is observed that the wall jet counter-clockwise vorticity layer is decayed and dissipated due to mixing and vanishes downstream at the suction slot where the CFWJ ends as shown in Fig. 20 (a). The vanishing of the counter-clockwise wall jet vorticity appears to create a sufficient mixing distance to place the CFWJ suction for maximizing the efficiency in the adverse pressure gradient. Fig. 20 (b) shows the vorticity contours of the optimum FPG injection Case 9, which has the 3 layers of counter-rotating vorticity layers thinner and shorter than those of Case 2 in APG.

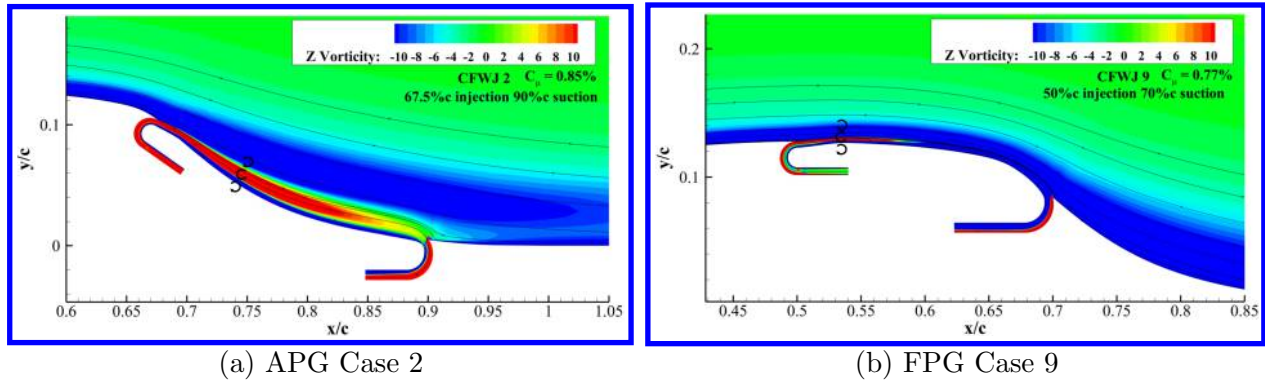


Figure 20: Spanwise vorticity for the optimum FPG and APG cases.

4.4.1 The Slots Effect with no Jets

The sizes of the injection slot and suction slot are very small compared with the scale of the flow field. They do not affect the flow field in a notable way as observed in the experiment [28] when the jet is turned off. However, it may be possible that the tripping effect of the slots could help to attach the flow. If that is the case, the energy expenditure analysis would be inaccurately attributed to the CFWJ effect.

To be certain that the slots of the CFWJ do not contribute to the flow attachment, the CFWJ Case 2 and Case 9 with no jets activated are simulated by sealing the injection inlet and suction outlet with no-slip wall boundary condition. The computed flow fields and flow separation patterns are virtually identical to those of the baseline flows shown in Fig. 5. This is evidenced by the surface C_p distributions of the CFWJ Case 2 and 9 with no jet compared with that of the baseline as shown in Fig. 21. They are basically the same except for the pressure spikes at 50% c and 90% c locations due to the open-slot effects. These results indicate that the CFWJ slots without jets have no benefit to the separation control of this hump flow. The flow attachment is fully attributed to the CFWJ effect.

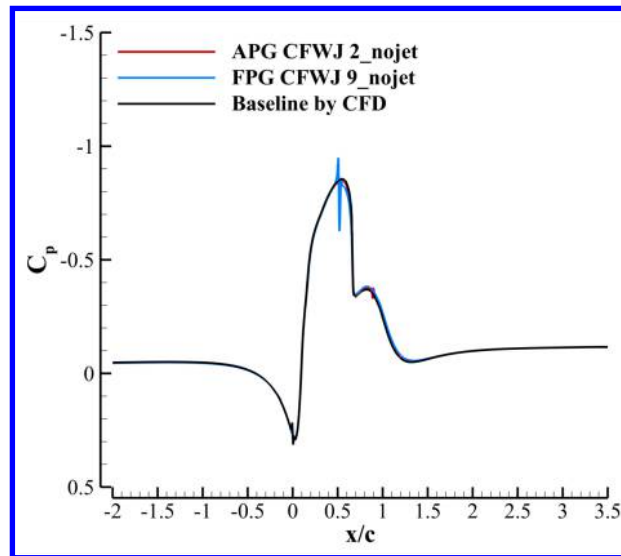


Figure 21: C_p distributions for the baseline and the optimum CFWJ cases with no jet.

4.5 CFWJ Comparison with Injection-only and Suction-only Flow Control

To form a high-efficiency self-contained ZNMF system, the injection and suction are used together by the co-flow wall jet. Other than satisfying the ZNMF role, it is important to answer the question raised in the introduction section whether there is a coupling effect of the injection and suction. In other words, is it more efficient and effective to use both injection and suction simultaneously than only to use injection or suction assuming ZNMF is not a concern? To answer these questions, the best way is to compare the effectiveness and efficiency of CFWJ and the AFC with only injection or suction. That is the purpose of this section.

4.5.1 Injection-only Flow Control

To have a rigorous comparison, the C_μ , slot location, orientation, and sizes are kept the same as the two optimum CFWJ cases, Case 2 and Case 9 shown in Table 6. To study the injection slot effect, the approach is to simply remove the suction slots of Case 2 and 9 and use the same injection slots at 50% c and 67.5% c (injection-only Case 1 and 2). As a trade study, one more injection location at 70% c (injection-only Case 3) adopted from the CFWJ Case 3 is added. Fig. 22 shows the three injection locations of the injection-only Case 1, 2, and 3. The injection-only Case 2 is indicated by an arrow to make the figure clear. The C_μ of 0.85% from the CFWJ Case 2 is used for all the injection-only cases for comparison.

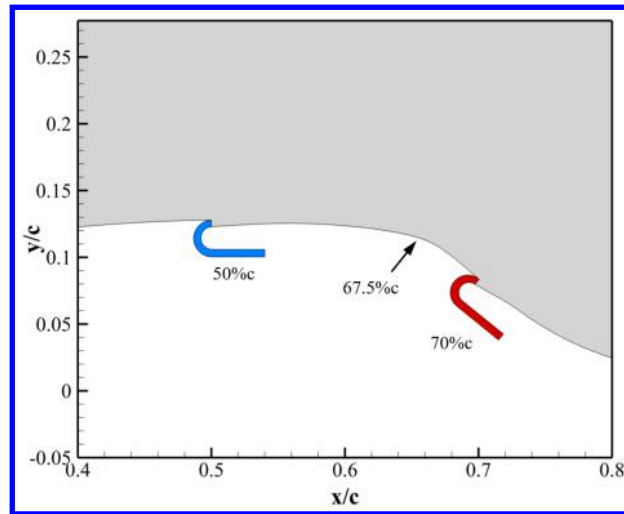


Figure 22: Injection location trade study (injection-only Case 1, 2 and 3).

Fig. 23 shows the Mach number contours of the three injection-only cases. The injection location at 50% c is the same as the location of the optimum CFWJ Case 9. However, without the downstream suction at 70% c as in the CFWJ Case 9, the injection-only case has a large flow separation as shown in Fig. 23 (a) even though the C_μ of 0.85% is higher than the C_μ of 0.77% of Case 9. When the injection slot is moved to 67.5% c in Fig. 23 (b), the same location as that of Case 2, a full flow attachment is achieved as the CFWJ Case 2 with the same C_μ . Further moving the injection slot downstream to 70% c , a small flow separation occurs upstream of the injection slot as shown in Fig. 23 (c), which is similar to the CFWJ Case 3 presented in Fig. 8.

Fig. 24 (a) shows the spanwise vorticity contours of the attached flow of the injection-only Case 2, which

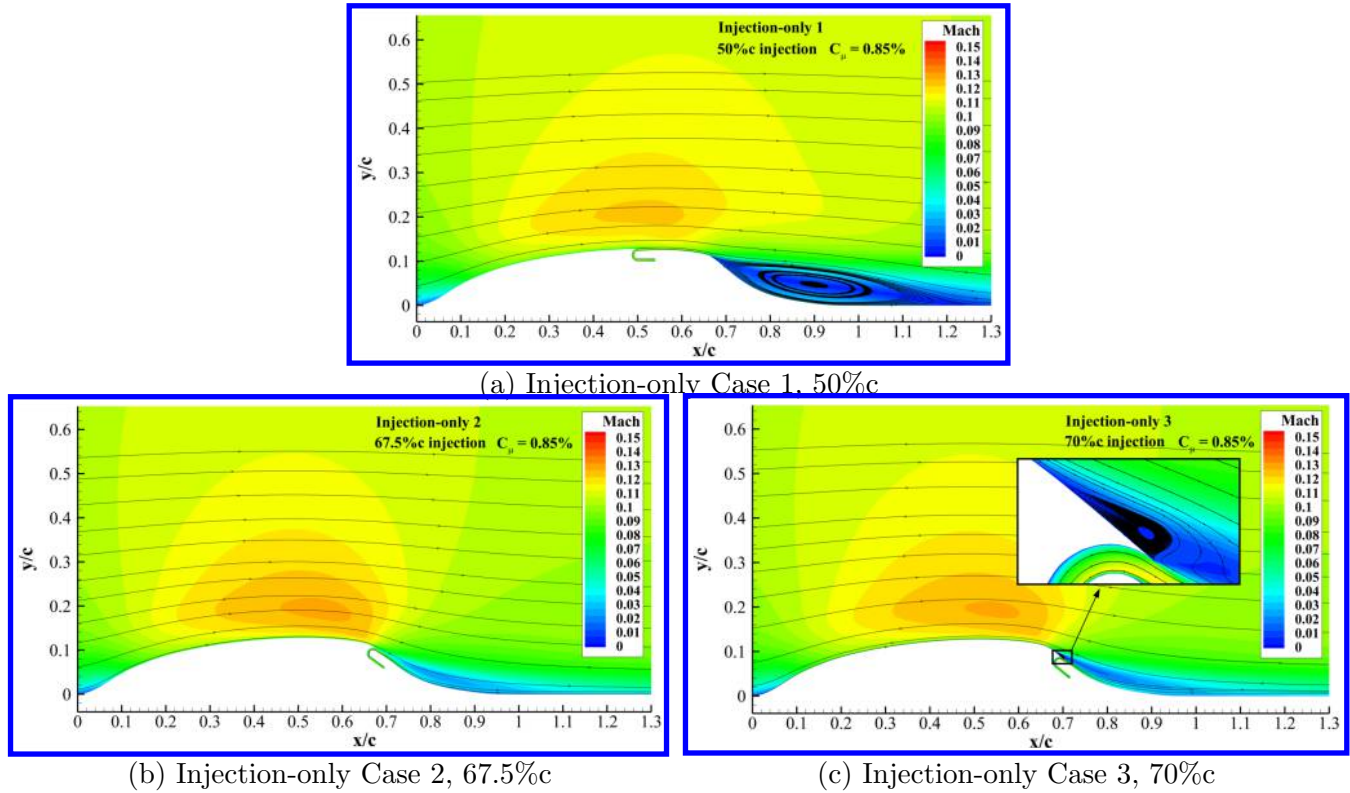


Figure 23: Injection-only trade study.

is very similar to the CFWJ Case 2 (with its suction slot shown by the dash lines in Fig. 24 (a)) that has the same injection location. Fig. 24 (b) - (f) also compare the velocity profiles (VPs) of the injection-only Case 2 with those of the CFWJ Case 2 and the baseline NASA hump at the same five streamwise stations labeled in Fig. 24 (a). The baseline NASA hump refers to the viscous results as shown in Fig. 5. At Station 1, upstream of the injection slot at $x/h=-3$ as shown in Fig. 24 (b), the velocity profiles of the CFWJ Case 2 and injection-only Case 2 are already energized by the induction effect of the downstream injection and are significantly fuller with higher kinetic energy than the baseline case. Downstream of the injection slot at $x/h=1$, the CFWJ Case 2 and injection-only Case 2 have almost identical wall jet VPs, which energize the boundary layer by mixing due to the high turbulent diffusion [38], whereas the baseline case is already separated with reversed velocities near the wall. As the flow reaches downstream to Stations 3 and 4, the wall jet VPs are dissipated due to the jet mixing and the VPs become typical turbulent boundary layer profiles. The CFWJ Case 2 has a slightly higher velocity than the injection-only Case 2 at Station 4 due to the downstream suction effect at 90% c . From Station 4 to 5, the flow passes the suction slot and the velocity profile of the injection-only Case 2 is slightly fuller than the CFWJ Case 2 at the near-wall region. This appears to be caused by the blockage of the suction slot as shown by the zoomed plot in Fig. 25. The flow bifurcates around the suction slot lip and affects the flow near the wall. However, the overall flow is more energized as shown by the stronger outer flow in Fig. 24 (f).

Overall, the injection-only Case 2 and CFWJ Case 2 have very similar velocity profiles at all the streamwise stations. This is because that the injection placed slightly downstream of the separation onset point plays a dominant role, which makes the suction not so important but more serves as the mass flow source for the injection. However, this does not mean that the injection-only is as effective as the CFWJ in general. When the injection is placed away from the critical position slightly downstream of the separation onset

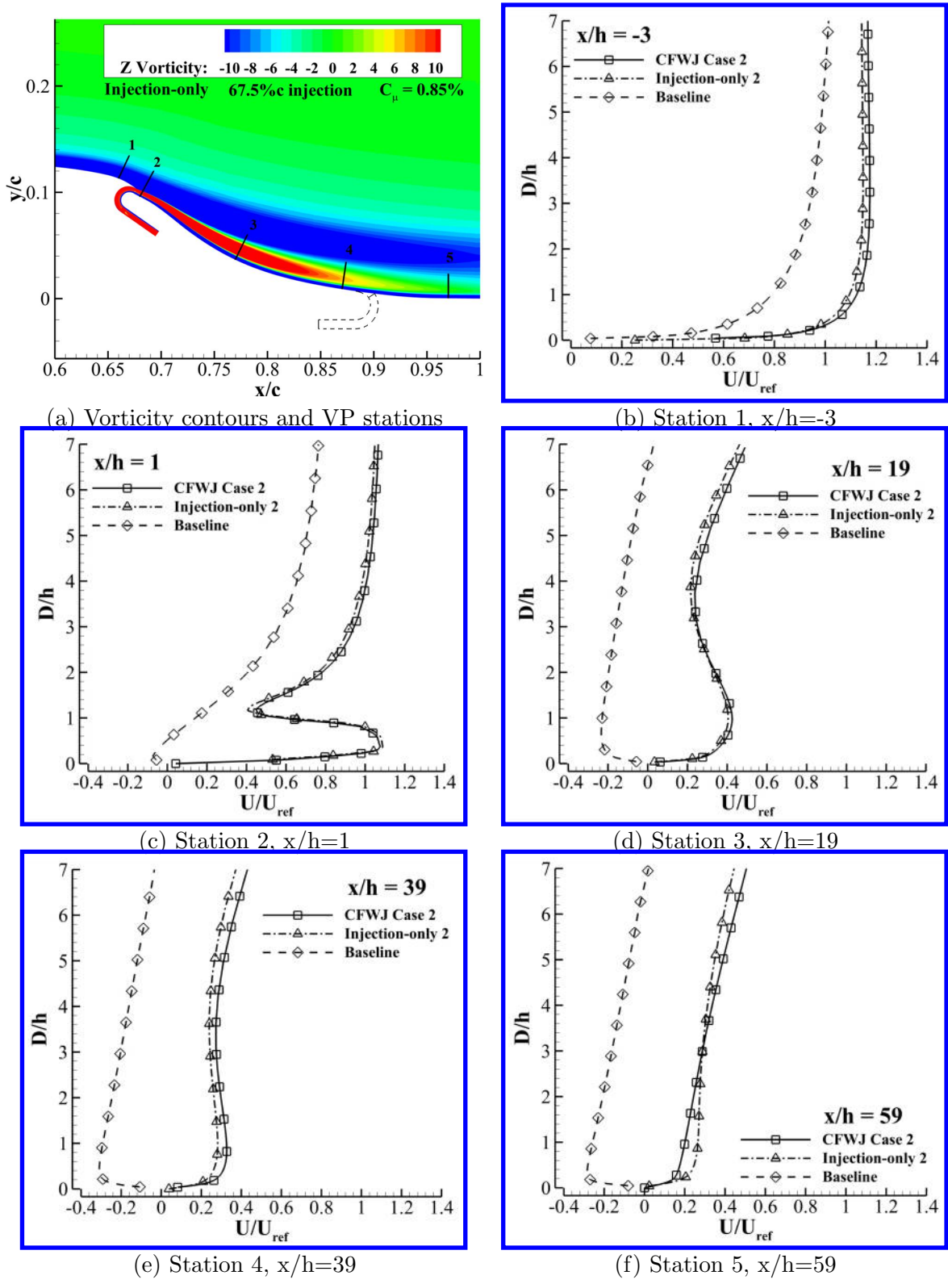


Figure 24: The spanwise vorticity of the injection-only Case 2 and the velocity profiles compared with the CFWJ Case 2.

point such as the injection-only Case 1, CFWJ is more effective. In practical applications, the optimal position to place the injection is usually not known *a priori*.

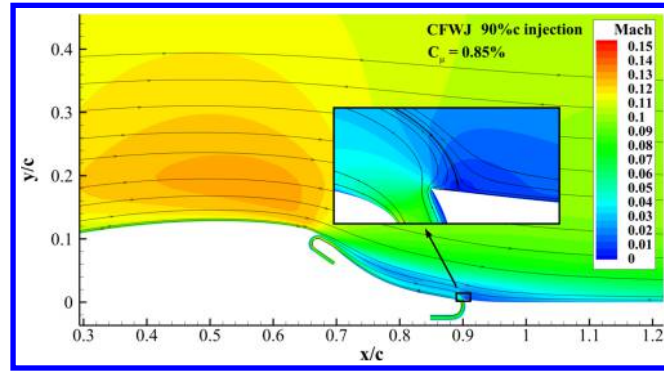


Figure 25: Streamlines of CFWJ Case 2 with zoomed view to show VP difference at Stations 4 and 5.

These injection-only results give the following conclusions: 1) When the injection slot is far upstream of the separation onset point such as 50%c, the injection-only flow control is not as effective as the CFWJ Case 9 that has a suction located at 70%c. 2) When the injection is located near the separation onset point as the CFWJ Case 2, the injection-only flow control with the same C_μ appears as effective as the CFWJ Case 2 in terms of flow attachment for this NASA hump configuration. However, even with the same control effectiveness, the CFWJ always has an advantage that it is ZNMF, which requires no external flow source.

The above conclusions are consistent with the observation for the CFWJ Case 2 and 7 in Section 4.2.2, for which the suction location at 85%c and 90%c yields the same CFWJ power coefficient since the CFWJ injection plays the dominant role to suppress the flow separation.

4.5.2 Suction-only Flow Control

Following the same approach, for the suction-only study, the injection slots of the CFWJ Case 2 and 9 are removed and the same suction slot located at 70%c and 90%c (suction-only Case 2 and 3) are used. To increase the samples of the trade study, the suction location of the CFWJ Case 8 is also adopted with the location at 67.5%c (suction-only Case 1) as indicated by an arrow in Fig. 26, which shows the suction locations of the three suction-only Case 1, 2, and 3. For the CFWJ cases, the momentum coefficient C_μ is calculated based on the injection jet. For the suction-only trade study, the C_μ has to be calculated based on the suction. The C_μ of 0.77% of the CFWJ Case 9 is thus recalculated based on its suction flow condition and the corresponding C_μ is 0.57%.

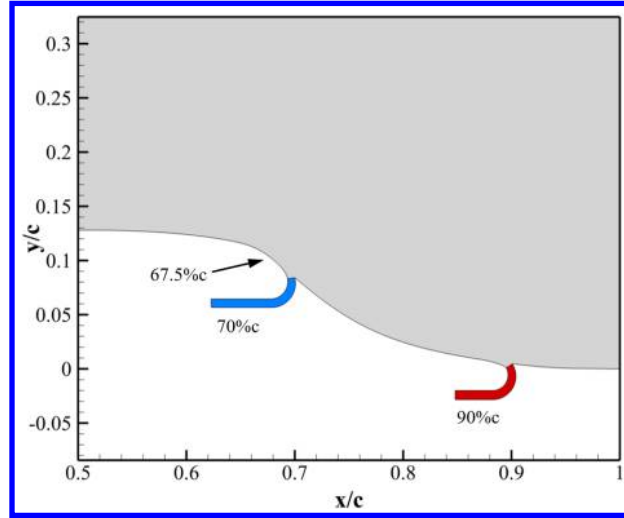


Figure 26: Suction location trade study (suction-only Case 1, 2 and 3).

Fig. 27 shows the Mach number contours of the three suction-only cases with the suction slot at $67.5\%c$, $70\%c$, and $90\%c$. The minimum C_μ to make the flow attached is 0.61% for suction-only Case 2 with the suction located at $70\%c$, 7% higher than 0.57% of the CFWJ Case 9. The suction-only Case 1 with the suction slot at $67.5\%c$ has a very small separation zone. The suction-only Case 3 with the slot at $90\%c$ (Fig. 27 (c)), the same as that of the CFWJ Case 2, has a large flow separation as shown in Fig. 27 (c) with C_μ of 0.57% . The separation remains even with the C_μ increased by 10 times.

The results of this suction-only study indicate that if the suction slot is placed at the optimal location downstream of the separation onset point at $70\%c$, it is able to attach the flow, but the required C_μ is higher than that of the CFWJ optimum Case 9. If the suction slot is off the optimal position, the CFWJ is much more effective since injection and suction all contribute to separation control.

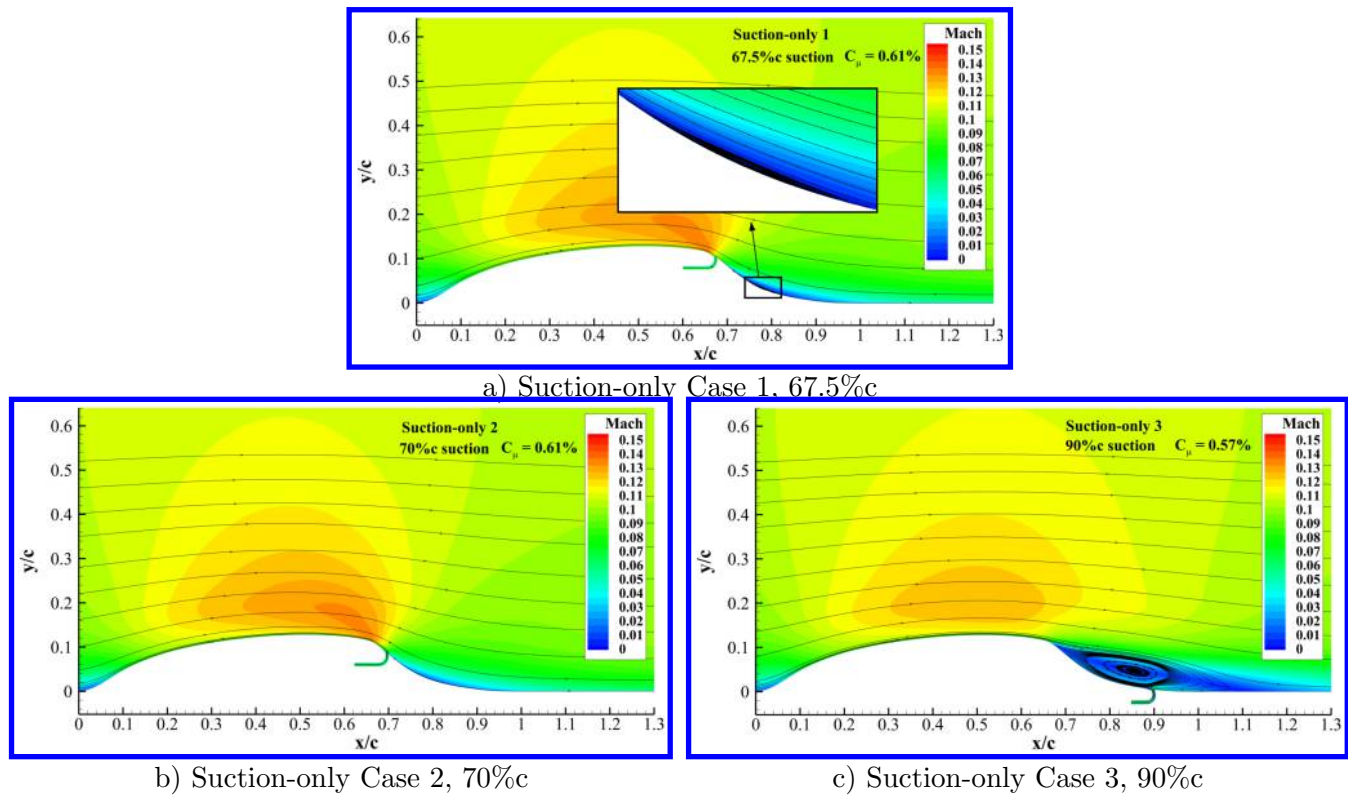


Figure 27: Suction-only trade study.

Fig. 28 shows the spanwise vorticity contours and the VPs of suction-only Case 2 compared with the CFWJ Case 9 (with its injection slot shown as the dash lines in Fig. 28 (a)) and the baseline hump with no flow control. A significant difference of the vorticity contours from that of Case 9 shown in Fig. 20 (b) is that the suction-only Case 2 has no counter-rotating vorticity layers within the boundary layer. There is only one clockwise vorticity layer as a typical wall boundary layer. In other words, the suction-only flow control does not generate a wall jet as defined by Launder and Rodi [35].

Fig. 28 (b) - (f) compare the velocity profiles at 5 streamwise stations for the baseline NASA hump, the CFWJ Case 9, and the suction-only Case 2. The station locations are measured from the suction slot and the distance is normalized by the suction slot height. As previously mentioned, the CFWJ Case 9 has the same suction slot location as the suction-only Case 2, but the C_{μ} of the CFWJ Case 9 is 7% smaller. At Station 1 located at 36 slot-height upstream of the suction slot, the VP of the suction-only is the fullest among the three profiles, fuller than that of the CFWJ Case 9 that has the injection located nearby downstream of Station 1. This indicates that the low-pressure effects of the streamwise suction can propagate far upstream in the subsonic field. The VP of the baseline hump is the weakest. However, the overall difference of the three VPs is not large at this location far upstream of the separation region. At Station 2, 27 slot-height upstream of the suction slot, the VP difference between the baseline and suction-only is significantly increased. A wall jet shape VP is formed by the CFWJ Case 9. At Station 3, the wall jet counter-clockwise vorticity layer of the CFWJ Case 9 diminishes due to the favorable pressure gradient [77]. At Station 4 immediately upstream of the suction slot, the VPs of the CFWJ and suction-only case are very similar without a wall jet protruding velocity profile shape. However, both the flow-controlled VPs are significantly energized compared with the separated VP of the baseline case. From Station 4 to 5, the flow decelerates in the adverse pressure gradient but remains attached for both the controlled flows,

whereas the baseline hump is severely separated. The CFWJ Case 9 has a C_μ of 7% smaller than that of the suction-only case, but it is the suction of the CFWJ Case 9 that plays the dominant role to remove the flow separation since the injection-only at the same upstream location does not remove the flow separation as shown in Fig. 23 (a).

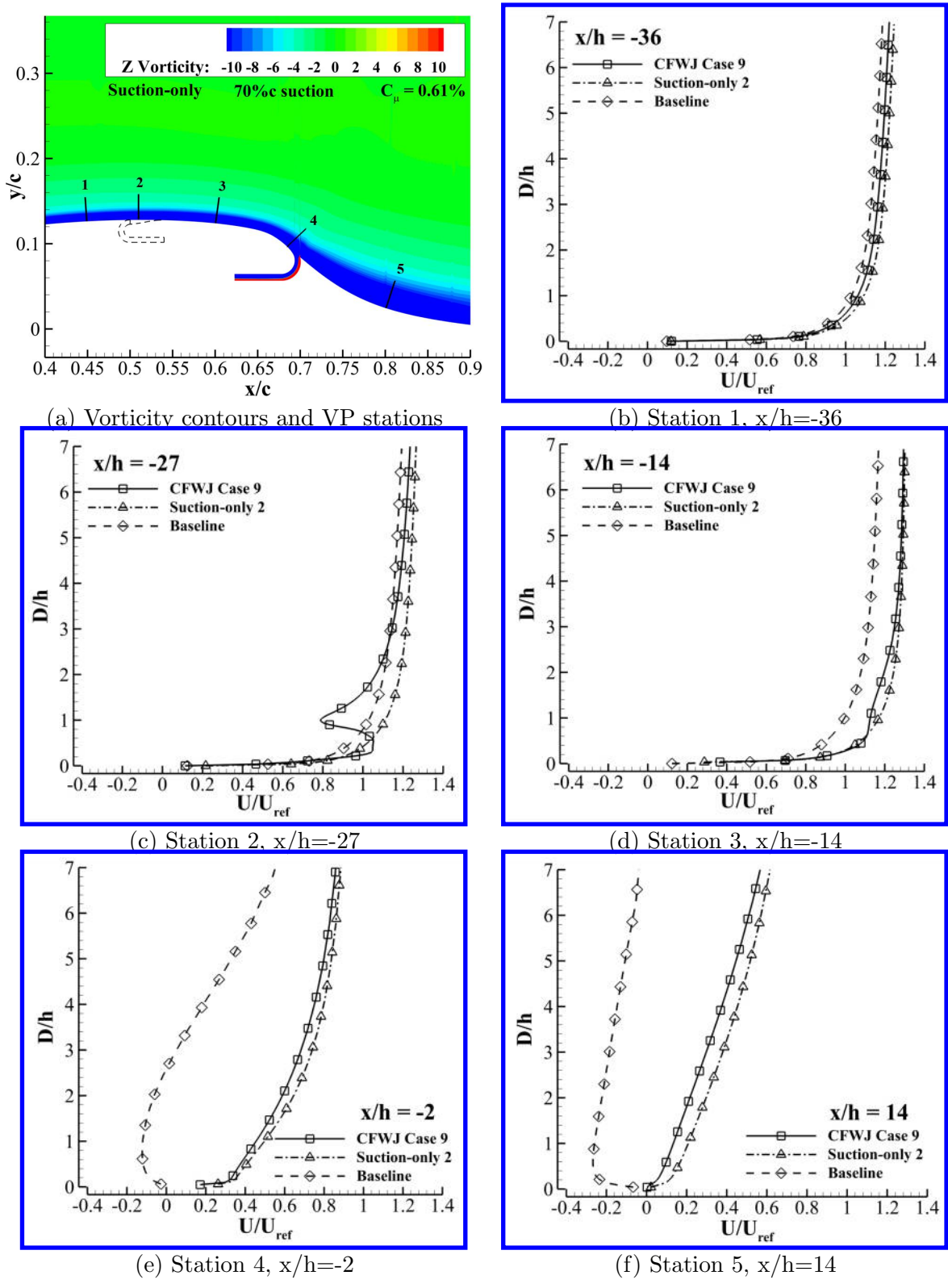


Figure 28: The spanwise vorticity of the suction-only Case 2 and the velocity profiles compared with the CFWJ Case 9.

By comparing the CFWJ cases with the injection-only case in Fig. 24 and the suction-only case in Fig. 28, it can be concluded that for CFWJ injection or suction, the one close to the separation onset plays the dominant role in suppressing flow separation. To maximize the CFWJ efficiency, there is a desirable distance between the injection and suction, which should be sufficiently long to allow a full mixing. For the injection or suction that is far away from the separation onset point, their effect on the separation suppressing is weak, but primarily serves as the source or sink to satisfy the mass conservation and achieve ZNMF. Both the injection-only and suction-only flow controls are effective if the slots are placed at the optimal position slightly downstream of the separation onset point. If they are off their optimal location, the injection-only or suction-only flow controls are far less effective than the CFWJ.

5 Conclusions

The energy expenditure of co-flow wall jet active flow control for the NASA hump is investigated numerically by trade studies of the injection and suction slot location. The injection-only and suction-only flow control methods are also studied and compared with the CFWJ. The followings are some conclusions obtained from the validated numerical simulation using 2D URANS with Spalart-Allmaras turbulence model.

1. The power required of a CFWJ fluidic actuator is exponentially determined by the total pressure ratio and linearly determined by the mass flow rate. To minimize the coefficient of power required P_c , reducing the total pressure loss is more important than reducing the mass flow rate. This principle applies to general AFC using fluidic actuators. The momentum coefficient C_μ should not be used to measure AFC energy consumption because a low C_μ may yield a high power required due to a high total pressure ratio even though the mass flow rate is low. The quantitative value of C_μ is also very different from P_c .
2. The CFWJ is effective in both adverse and favorable pressure gradients but is more energy efficient to be used in adverse pressure gradients due to lower flow velocity, lower entropy increase, and enhanced mixing. As a result, the APG generates less main flow loss and thus requires a lower total pressure ratio of the CFWJ actuators.
3. To minimize the CFWJ energy expenditure, it is important to provide a sufficiently long distance between the injection and suction for thorough mixing and energy transfer in adverse pressure gradients. It is observed in this study that the distance with the counter-clockwise vorticity layer vanishing is sufficient.
4. Both the tangential injection and streamwise suction of CFWJ are effective to suppress flow separation respectively. It is most efficient to apply the injection or the suction near the flow separation onset point at a slightly downstream location in adverse pressure gradients. For this NASA hump, it is observed that the optimal suction location near the separation onset point is a little downstream of the optimal injection location.
5. Placing the injection of the CFWJ in the deep flow separation region is not able to remove the flow separation, but placing suction of CFWJ in the deep flow separation region is able to remove the flow separation attributed to the coupling effect of the upstream CFWJ injection.
6. For the injection-only or suction-only flow control, when the slots are at the optimal location downstream of the flow separation onset point, the injection-only flow control is as effective as the CFWJ

for suppressing flow separation. The suction-only flow control needs a momentum coefficient 7% higher than that of the CFWJ to suppress the flow separation. However, if the slots are placed off the optimal position, the CFWJ flow control is much more effective and efficient than the injection-only or suction-only flow control.

6 Acknowledgment

The authors would like to acknowledge the computing resource provided by the Center of Computational Sciences at the University of Miami.

Disclosure: The University of Miami and Dr. Gecheng Zha may receive royalties for future commercialization of the intellectual property used in this study. The University of Miami is also an equity owner in CoFlow Jet, LLC, licensee of the intellectual property used in this study.

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