AIAA Paper 2004-1118

Boundary Layer Loss Mechanism and Justification of Wall Functions for Turbulence Modeling

Ge-Cheng Zha

Dept. of Mechanical Engineering
University of Miami
Coral Gables, Florida 33124
E-mail: zha@apollo.eng.miami.edu

Objective:

- Study the boundary layer (BL) loss mechanism for internal flows

- Examine the applicability of wall functions to predict internal flow loss

Introduction

• Loss prediction is important for internal flows,
which is an integral across the boundary layer

• Surface force prediction is important for external flows,
which is not an integral across the boundary layer

• Entropy increase is the measure of internal flow loss

$$\Delta S = -Rln\frac{Pt_2}{Pt_1} \tag{1}$$

Compressor Efficiency

$$\eta = \frac{Ideal\ Work}{ActualWork} = 1 - \frac{T_{t2}\Delta S}{H_{t2} - H_{t1}} \tag{2}$$

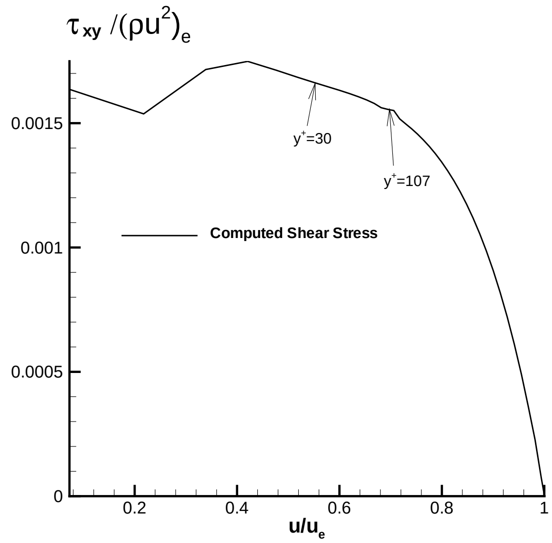

Figure 1: The distribution of entropy creation due to shear stress work across the boundary layer.

Denton's theory on entropy creation:

- Total rate of entropy creation across the BL

$$\dot{S}_a = \frac{d}{dx}\int_0^{\delta}(\rho V_x(s - s_\delta))dy = \int_0^{\delta}\frac{1}{T}\tau_{xy}dV_x \tag{3}$$

- Local rate of entropy creation within the BL

$$\dot{S}_v = \frac{1}{T}\tau\frac{dV}{dy} \tag{4}$$

Question:

- For turbulent BL, can wall function boundary conditions predict the loss correctly?

- Wall functions: based on the law of the wall

$$u^+ = \frac{1}{k} ln y^+ + B \tag{5}$$

$y^+ \approx 30$ to 200

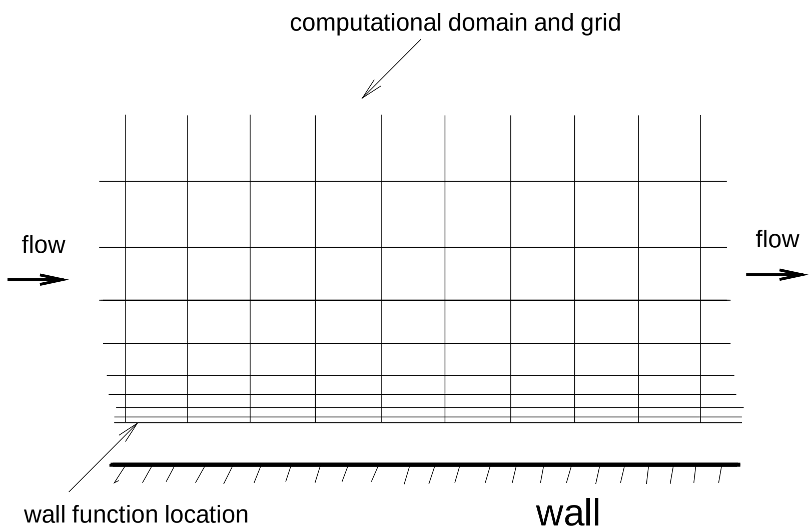

Figure 2: A sketch of the wall function location and computational domain

With Wall Functions:

- Total rate of entropy creation:

$$\dot{S}_a = \int_0^{\delta} \frac{1}{T}\tau_{xy} dV_x = \int_0^{y_{wallfunction}} \frac{1}{T}\tau_{xy} dV_x + \int_{y_{wallfunction}}^{\delta} \frac{1}{T}\tau_{xy} dV_x \quad (6)$$

$$\dot{S}_a = \int_{y_{wallfunction}}^{\delta} \frac{1}{T}\tau_{xy} dV_x \quad (7)$$

- Will eq.7 miss most of the entropy creation across BL?

- Answer: Yes.

- Can wall functions be used?

Boundary Layer Loss Mechanism

Turbulent BL eq. for flat plate:

Continuity equation:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{8}$$

X-momentum equation:

$$\rho(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \tag{9}$$

Y-momentum equation:

$$\frac{\partial p}{\partial y} \approx 0 \tag{10}$$

Energy equation:

$$\rho(u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y}) = u\frac{\partial p}{\partial x} - \frac{\partial q}{\partial y} + \tau_{xy}\frac{\partial u}{\partial y} \tag{11}$$

Thermodynamic relation:

$$dh = Tds + \frac{1}{\rho}dp \tag{12}$$

Boundary Layer Loss Mechanism

The energy eq.(local rate of entropy creation) becomes:

$$\frac{\partial s}{\partial l}d\dot{m} = \frac{1}{T}(\tau_{xy}\frac{\partial u}{\partial y} - \frac{\partial q}{\partial y})dy \tag{13}$$

or

$$\frac{\partial s}{\partial l}d\dot{m} = \frac{1}{T}(\tau_{xy}du - dq) \tag{14}$$

The total rate of entropy creation

$$\dot{S}_a = \int_0^{\dot{m}_\delta} \frac{\partial s}{\partial l}d\dot{m} = \int_0^\delta \frac{1}{T}(\tau_{xy}\frac{\partial u}{\partial y} - \frac{\partial q}{\partial y})dy \tag{15}$$

or

$$\dot{S}_a = \int_0^{\dot{m}_\delta} \frac{\partial s}{\partial l}d\dot{m} = \int_0^\delta \frac{1}{T}(\tau_{xy}du - dq) \tag{16}$$

Entropy creation

For adiabatic wall, $q_w = q_\delta = 0$, assume $T \approx C$

The total rate of entropy creation

$$\dot{S}_a = \int_0^{\dot{m}_\delta} \frac{\partial s}{\partial l} d\dot{m} = \int_0^\delta \frac{1}{T} \tau_{xy} du \tag{17}$$

• This is the same as Denton's conclusion for the total rate of entropy creation.

• The local rate of entropy creation:

$$\frac{\partial s}{\partial l} d\dot{m} = \frac{1}{T}(\tau_{xy} du - dq) \neq \frac{1}{T} \tau_{xy} du \tag{18}$$

• Denton's local rate of entropy creation is incorrect.

Entropy creation in a Boundary Layer: Solution validation

A duct, Inlet M=0.2, $Re_h = 10^5$,
$k - \epsilon$ model integrating to wall
$y_1^+ = 1.8$, $Re_\theta = 3584.5$, $H = 1.34$

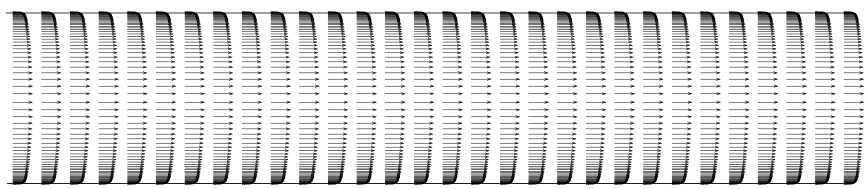

Figure 3: **Velocity vector field of the turbulent boundary layer in the duct near exit,** $M_{freestream} = 0.2$

Entropy creation in a Boundary Layer: Solution validation

The dissipation coefficient, Cd,

$$Cd = \frac{T\dot{S}_d}{\rho_e u_e^3} \tag{19}$$

where

$$\dot{S}_d = \int_0^\delta \frac{1}{T}\tau_{xy}\frac{du}{dy}dy \tag{20}$$

Computed numerical value: $Cd = 0.001463$, agree well with empirical correlation [Schlichting,1966]:

$$Cd = 0.0056 Re_\theta^{-1/6} = 0.001432 \tag{21}$$

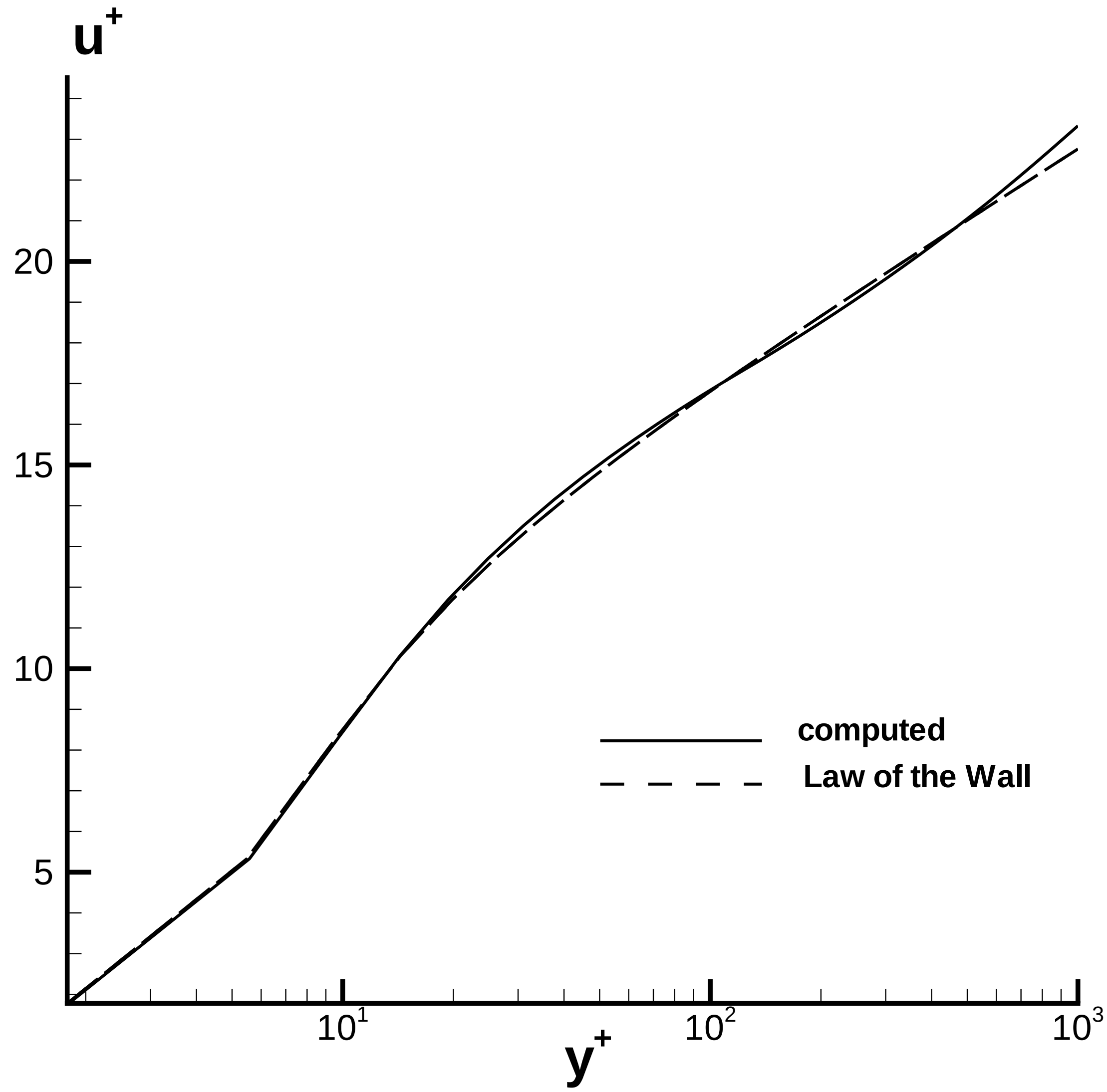

Figure 4: **Computed velocity profile in the inner layer compared with the law of the wall.**

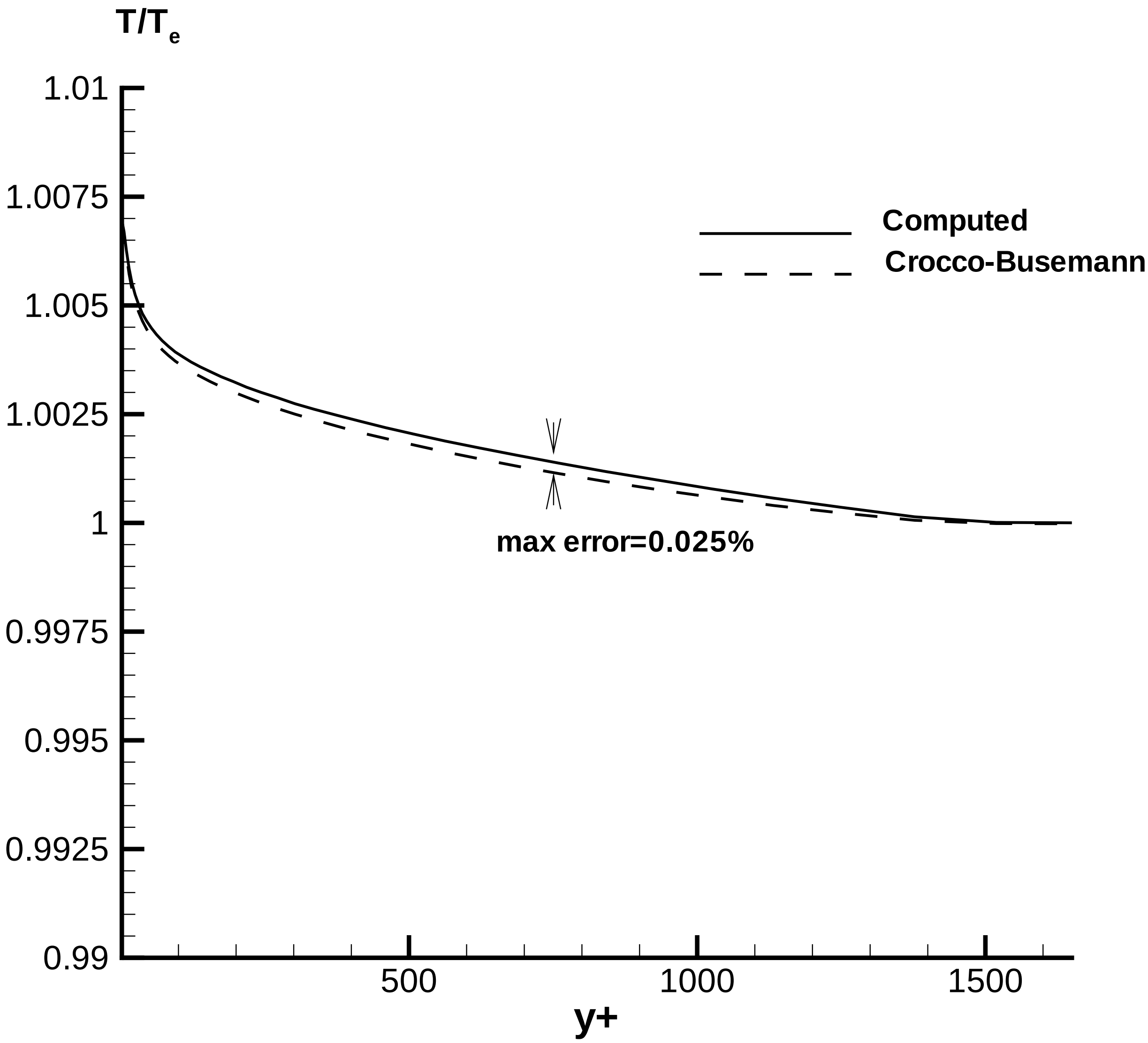

Figure 5: **Computed temperature profile of the turbulent boundary layer compared with Crocco-Busemann solution.**

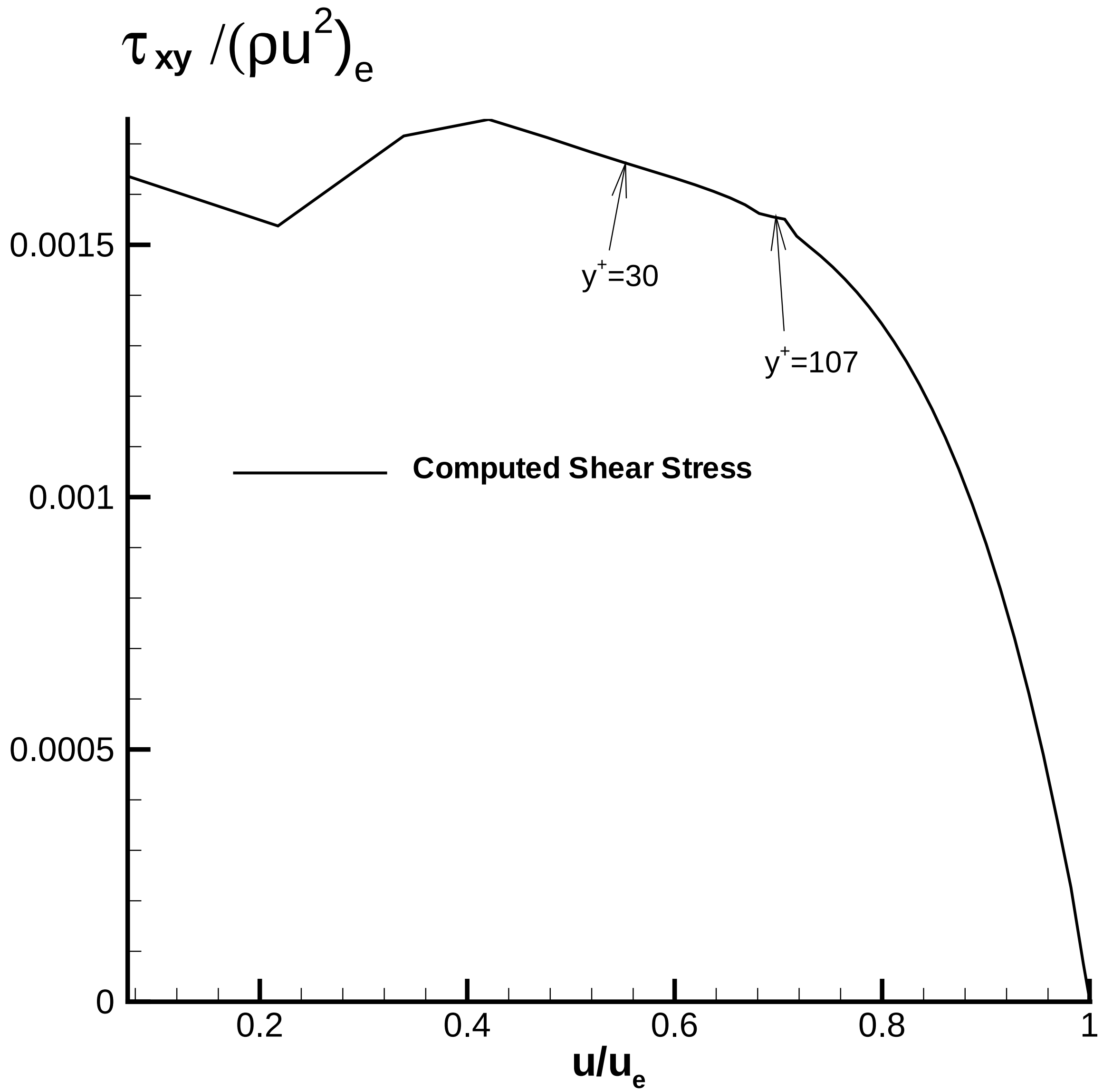

Figure 6: **The distribution of entropy creation due to shear stress work across the boundary layer.**

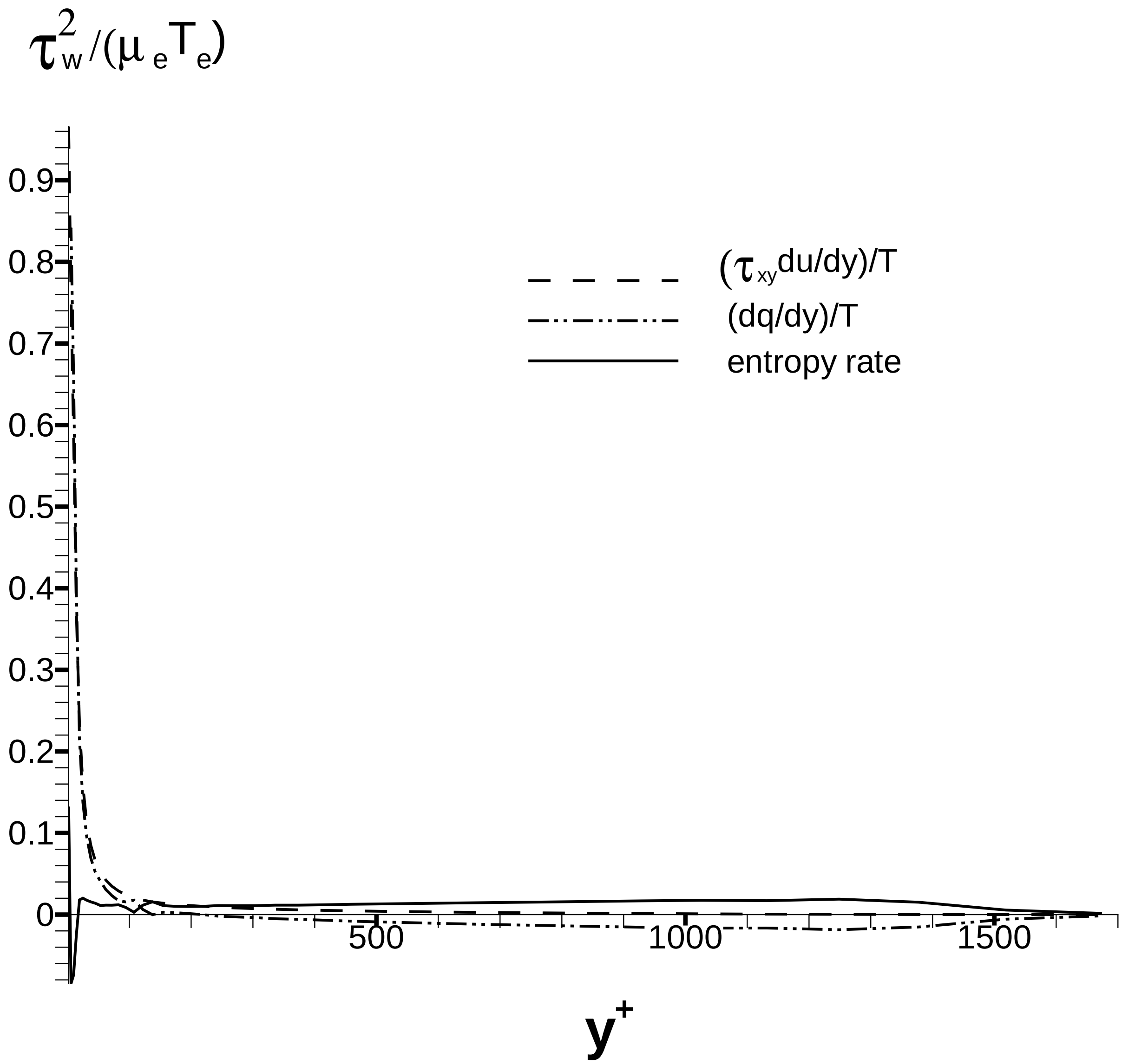

Figure 7: **The distribution of entropy creation across the boundary layer for individual terms and their resultant.**

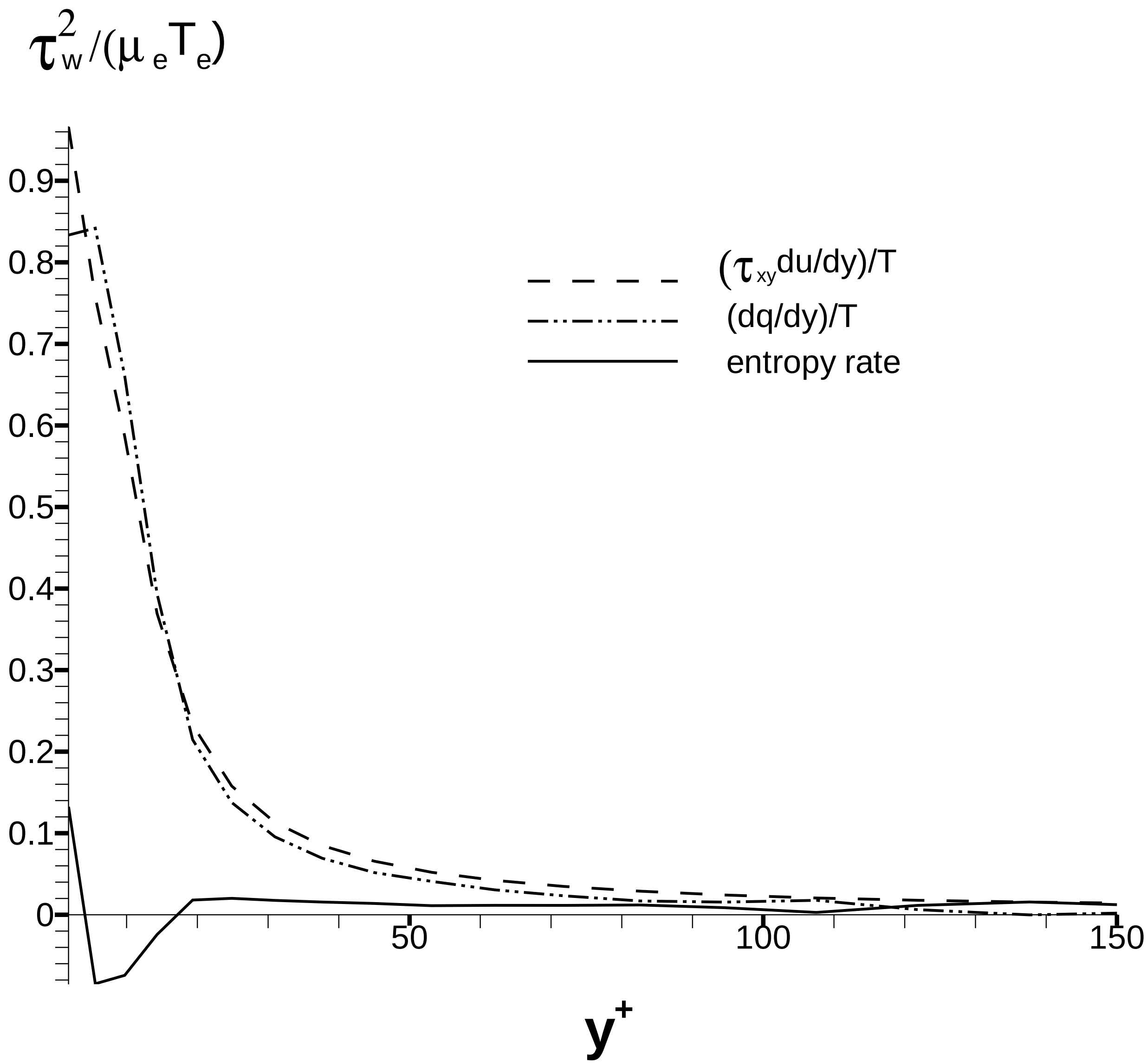

Figure 8: **The distribution of entropy creation near the wall for individual terms and their resultant.**

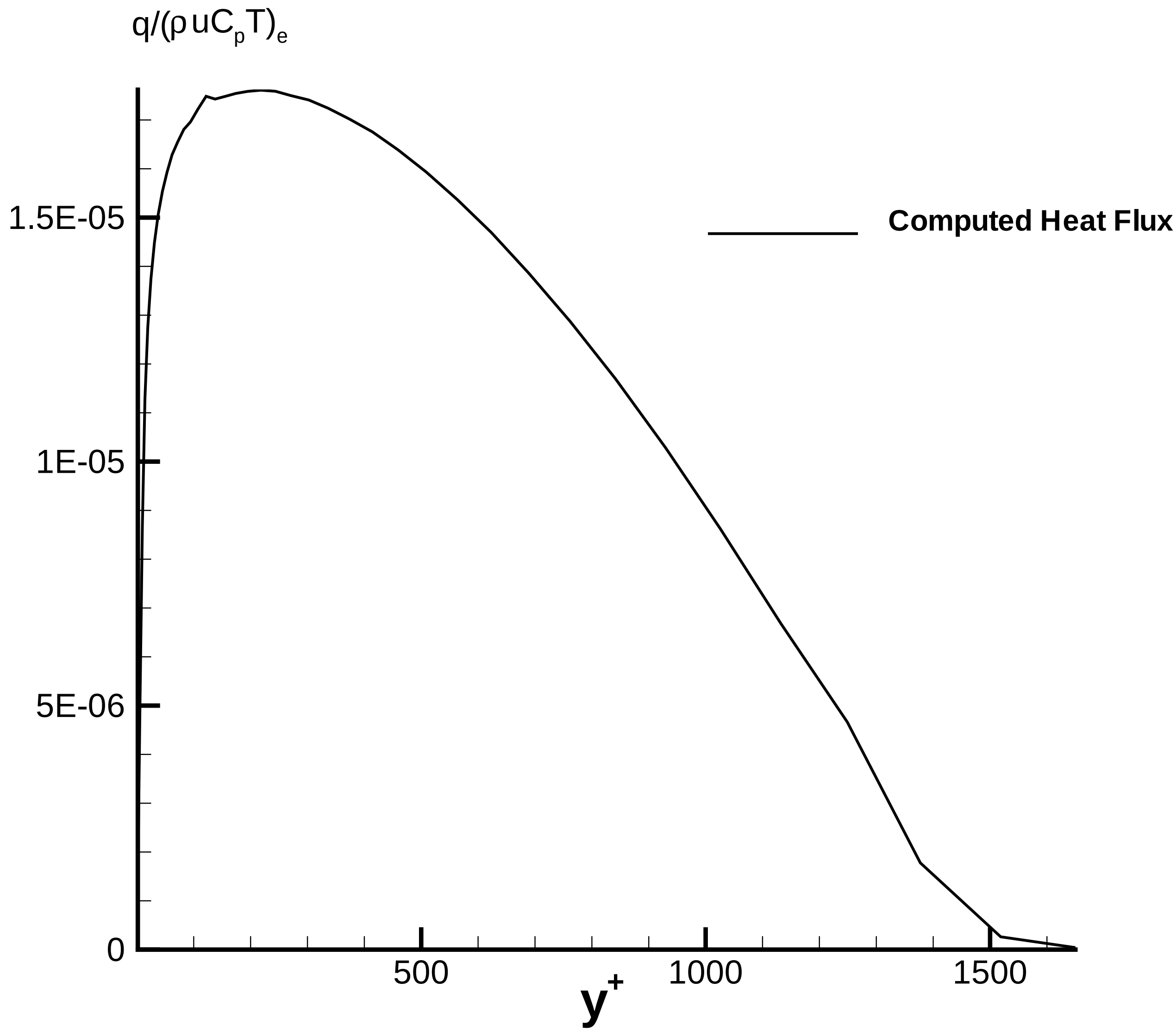

Figure 9: **Heat flux distribution for the duct adiabatic boundary layer.**

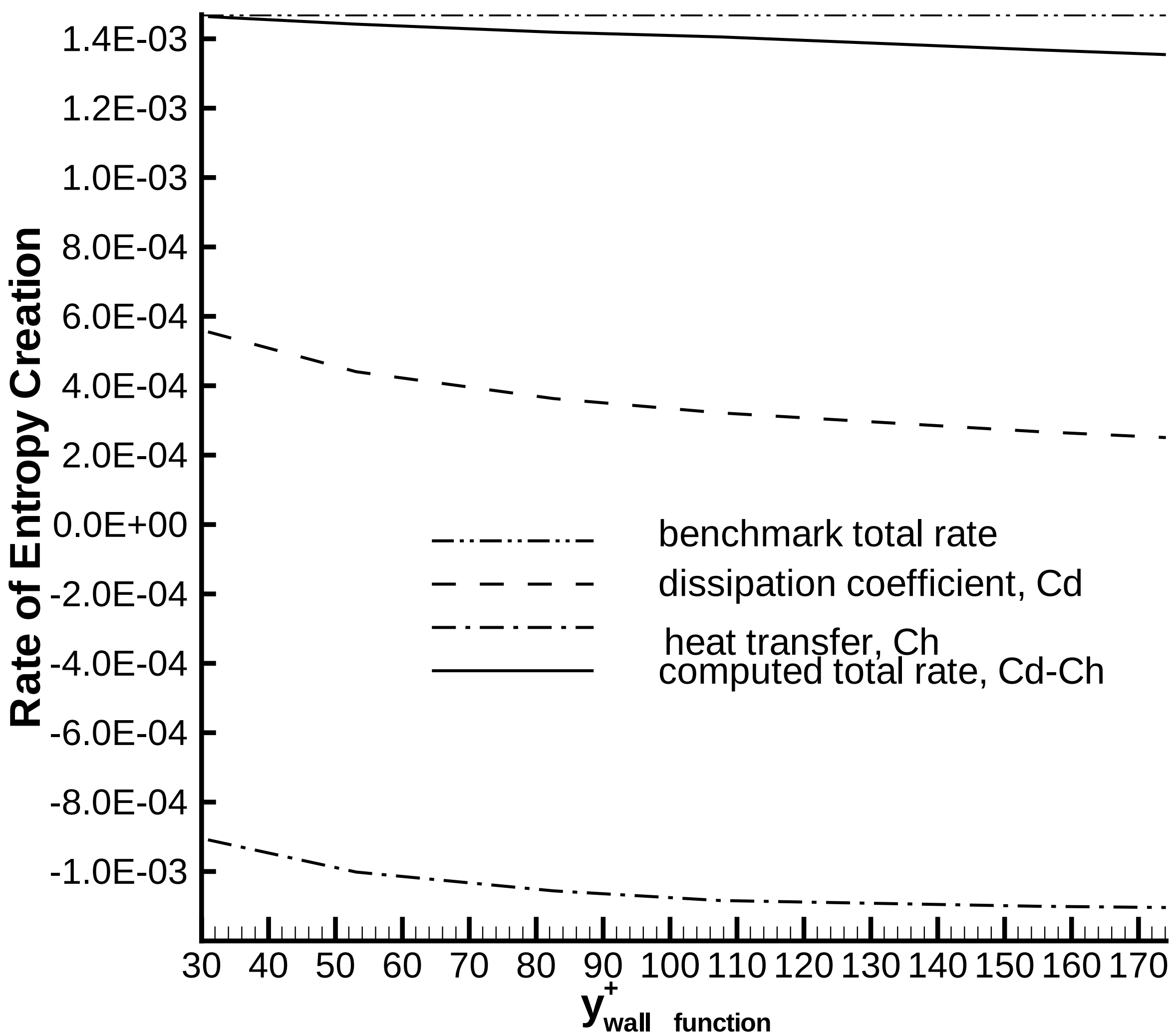

Figure 10: **Entropy creation rate computed at different wall function location.**

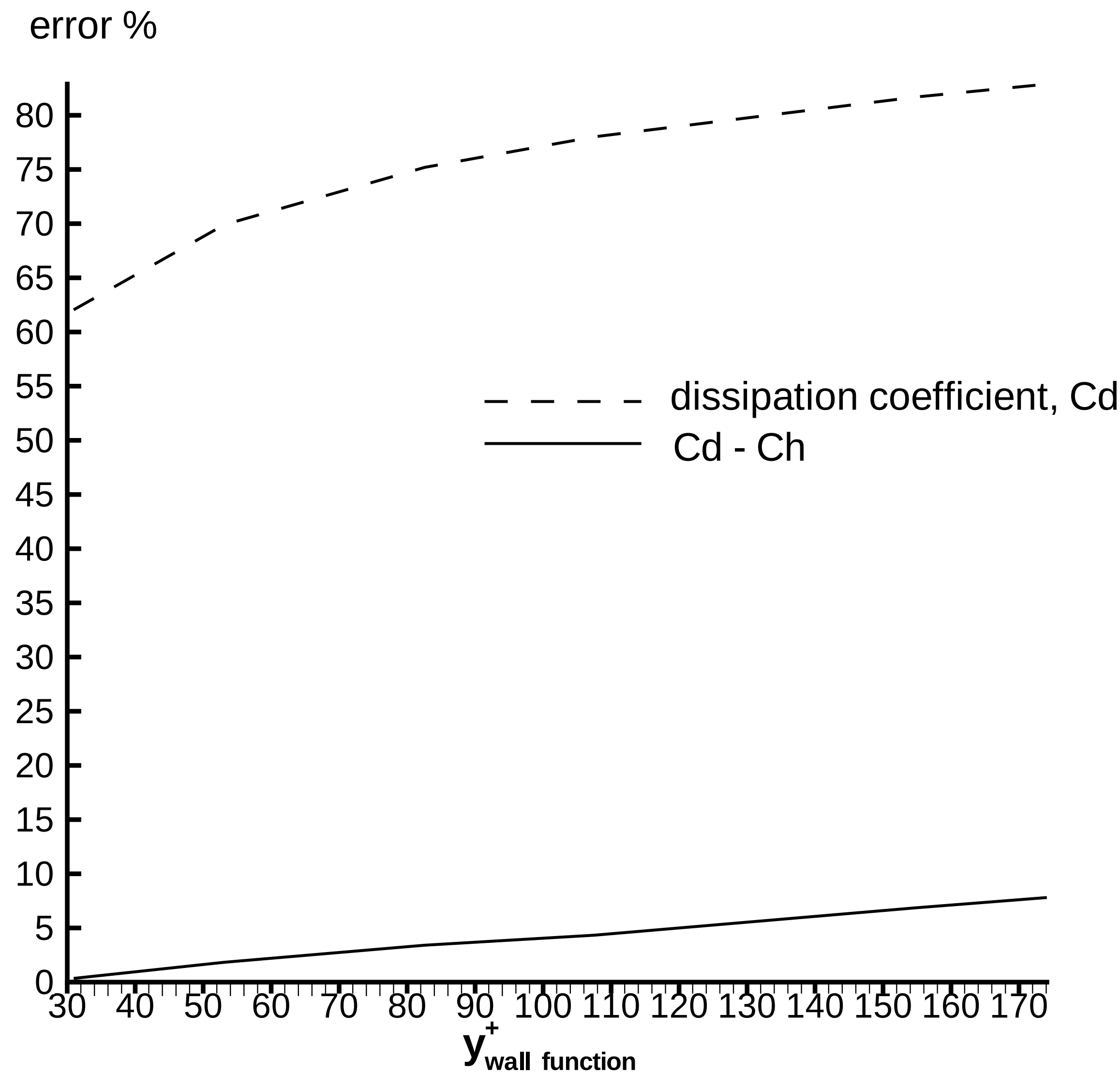

Figure 11: **Error of the total entropy creation rate computed at different wall function location.**

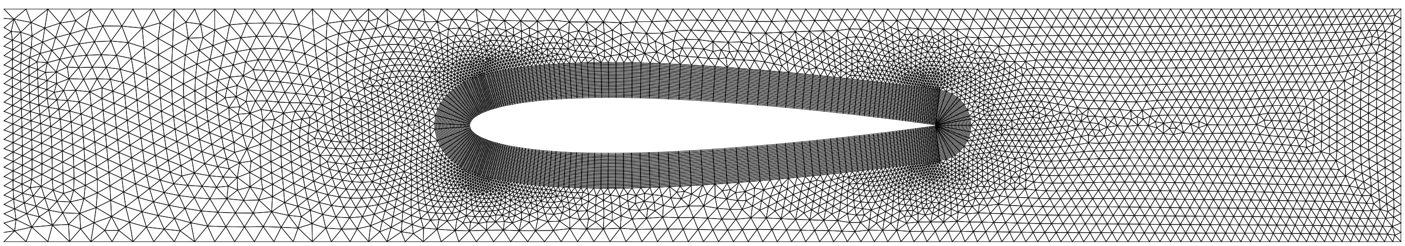

Figure 12: **Mesh for the NACA0012 cascade solution integrating to the wall.**

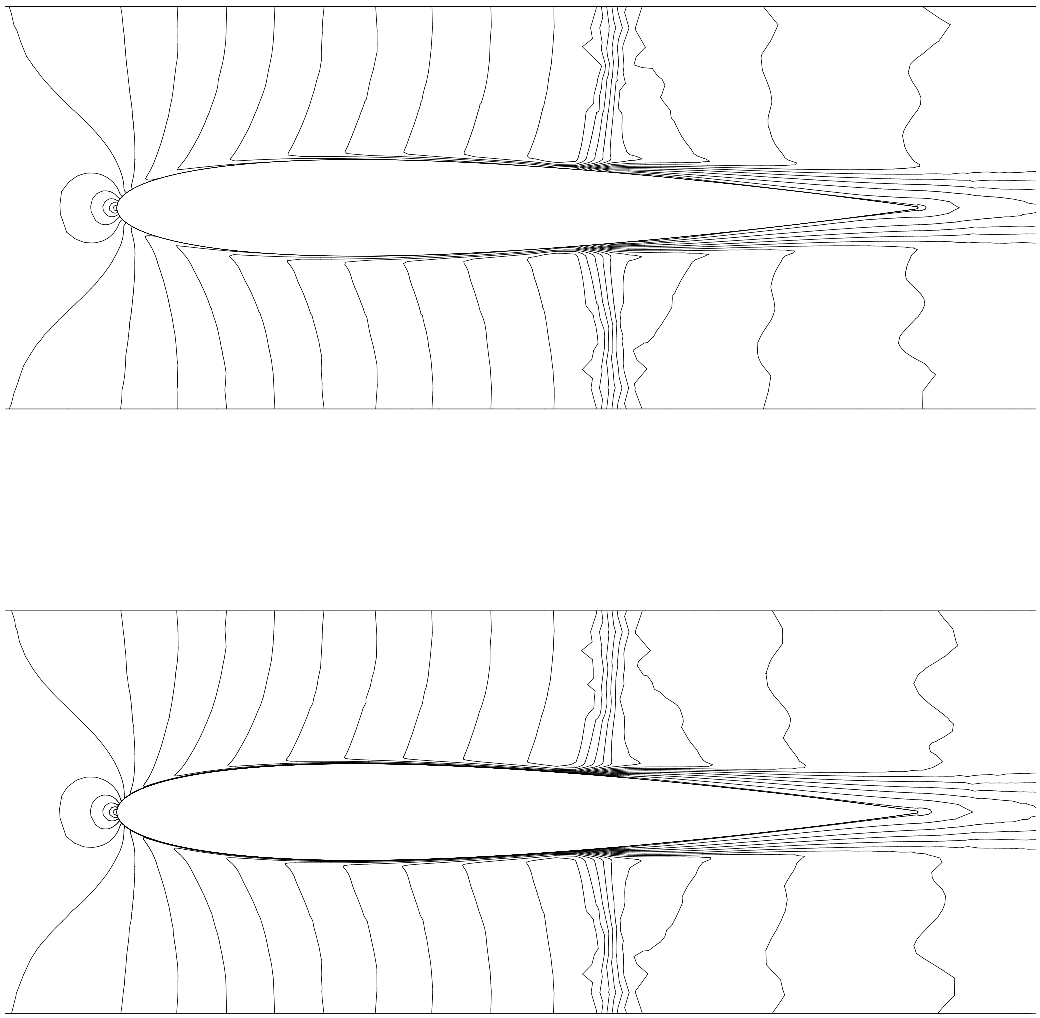

Figure 13: **Mach number contours of the cascade, top: using wall functions; bottom: integrating to the wall.**

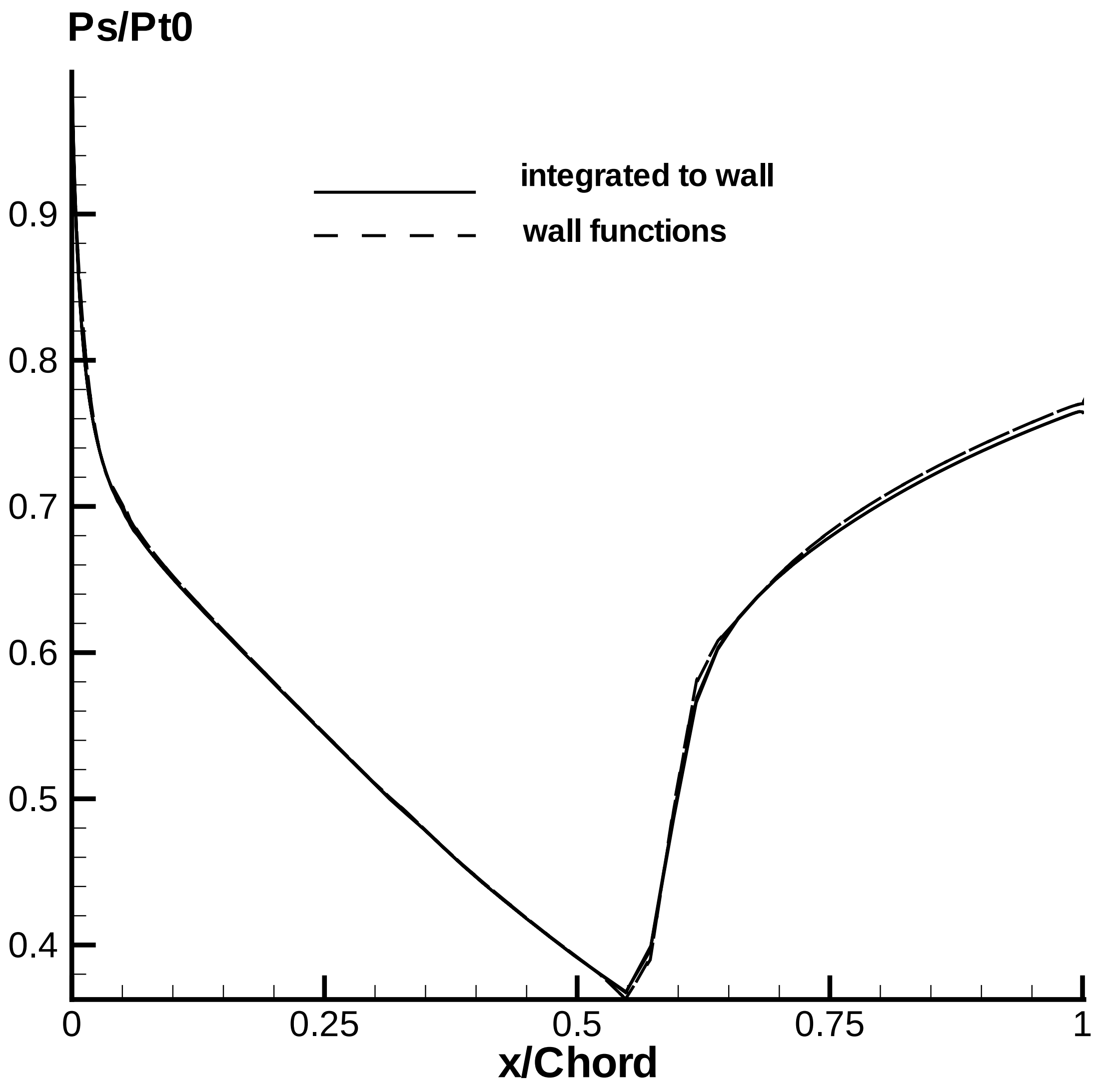

Figure 14: **Static pressure distributions of the cascade flow solutions using wall functions and integrating to the wall.**

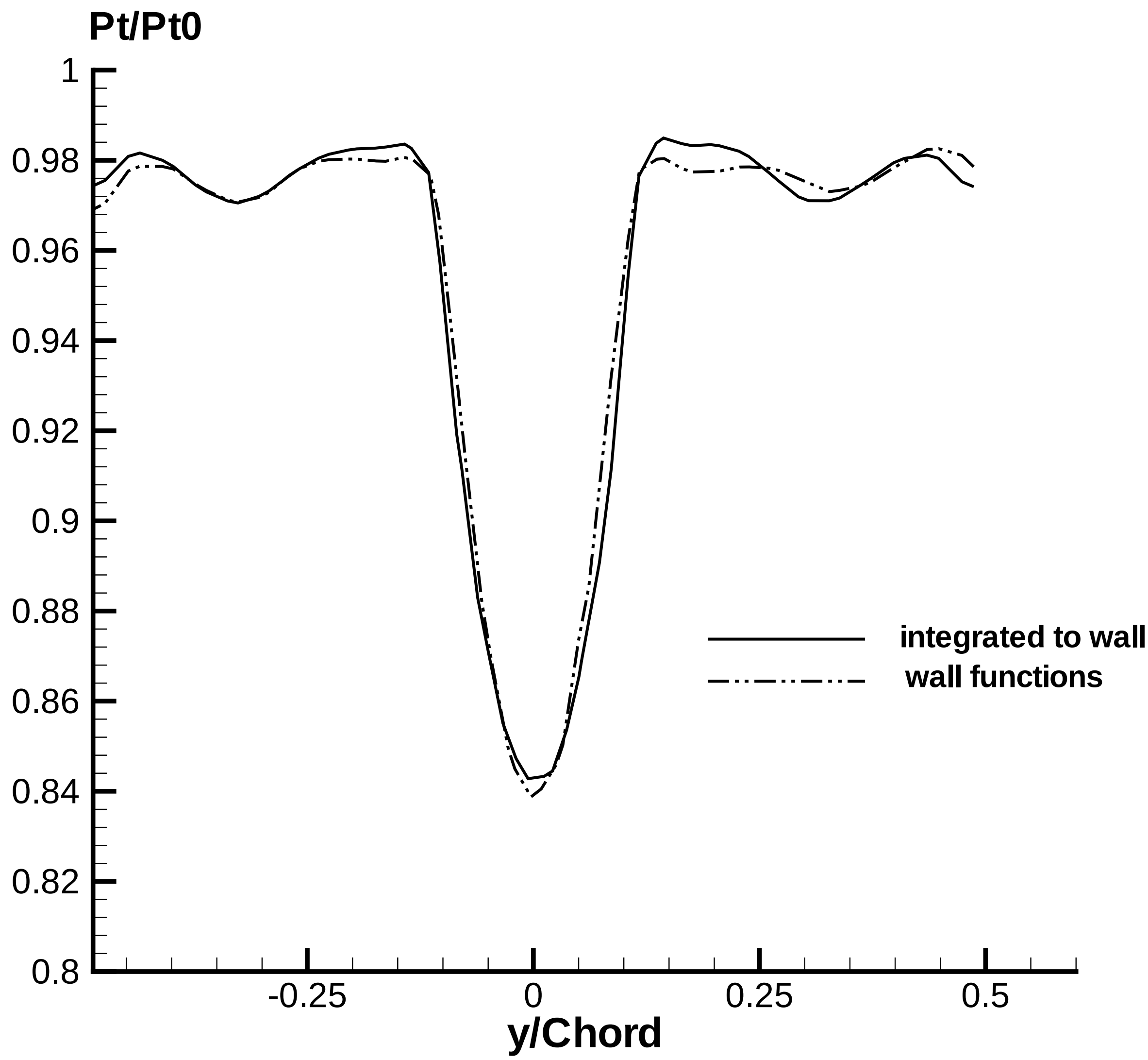

Figure 15: **Total pressure distributions in the wake region for the cascade flow solutions using wall functions and integrating to the wall.**

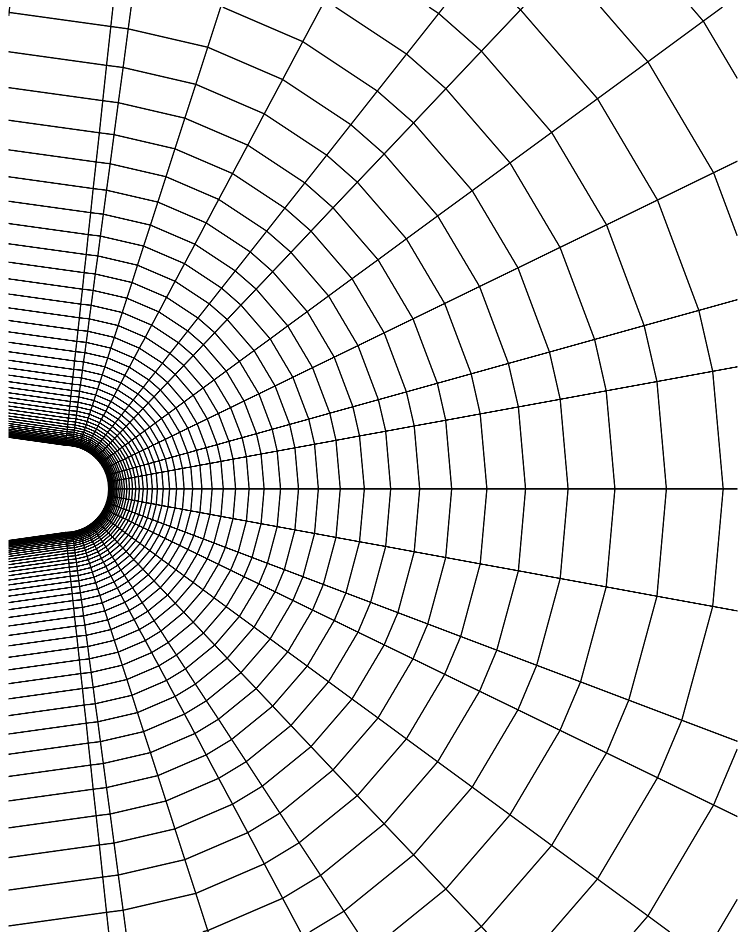

Figure 16: **esh in the trailing edge region for the solution integrating to the wall.**

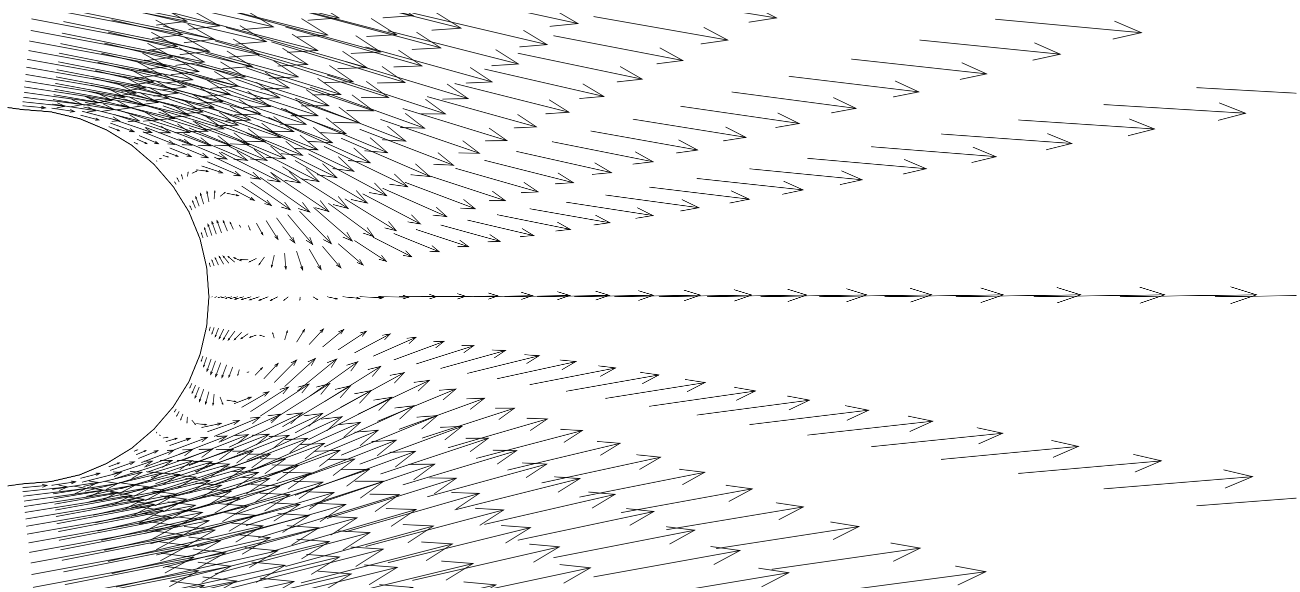

Figure 17: **Velocity vector field in the trailing edge region for the solution integrating to the wall.**

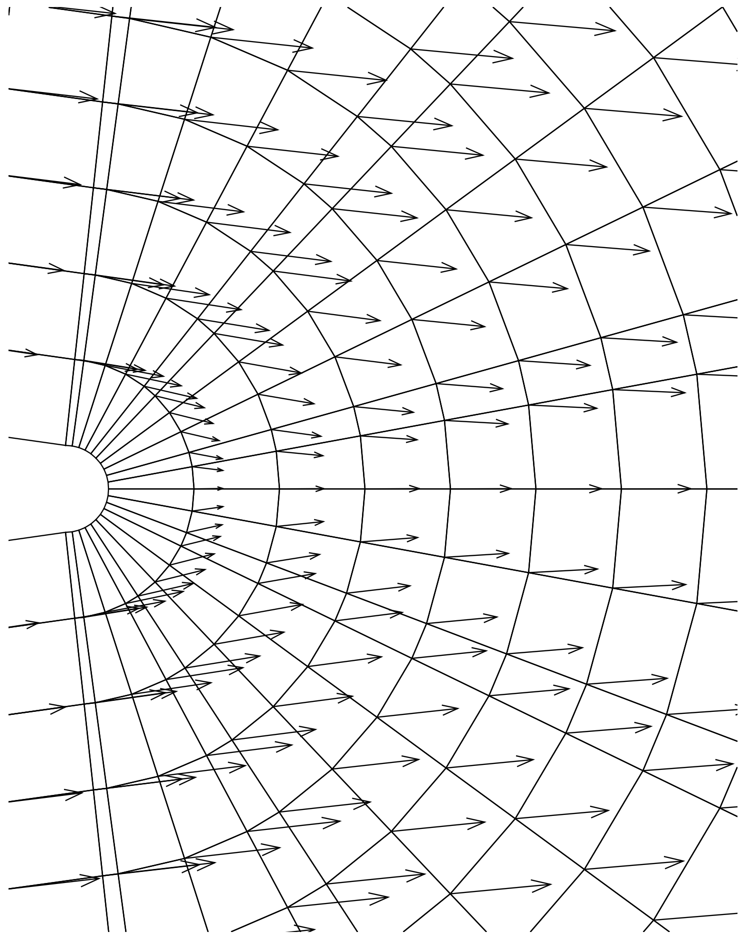

Figure 18: **Mesh and velocity vector field in the trailing edge region for the solution using wall functions.**

Conclusions:

• For an adiabatic turbulent boundary layer, the entropy creation within the boundary layer has two sources:

1) Shear stress work
2) Heat Flux gradient

• The entropy creation is fairly uniform across the boundary layer

• The previous theory that the entropy is mostly created in the inner layer is incorrect.

• The error to predict entropy creation using wall functions for turbulent boundary layer is small.

• There is a balance point between the shear stress work and heat flux gradient located at about $y^+ = 25 - 30$, where the wall functions will give correct entropy creation results.