AIAA Paper 2006-0697, 01/10/2006

Numerical Simulation of 3-D Wing Flutter with Fully Coupled Fluid-Structural Interaction

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Background

- Important Issue: Aircraft Wing and Turbomachinery Blade Flutter
- Fully coupled fluid-structure model is necessary to capture the nonlinear flow phenomena

Objective

- Develop High Fidelity Predicting Tool for Wing/Blade Flutter

Numerical Strategy

- 3D Time Accurate RANS
- Dual Time Stepping, Implicit Gauss-Seidel Iteration
- Low Diffusion Upwind Scheme
- 2nd Order Accuracy in Space and Time
- Baldwin-Lomax Turbulence Model
- Enforce Geometry Conservation Law

Governing Equations: 3D RANS

$$\frac{\partial \mathbf{Q}'}{\partial \tau} + \frac{\partial \mathbf{Q}'}{\partial t} + \frac{\partial \mathbf{E}'}{\partial \xi} + \frac{\partial \mathbf{F}'}{\partial \eta} + \frac{\partial \mathbf{G}'}{\partial \zeta} = \frac{1}{Re}\left(\frac{\partial \mathbf{E}'_{\mathbf{v}}}{\partial \xi} + \frac{\partial \mathbf{F}'_{\mathbf{v}}}{\partial \eta} + \frac{\partial \mathbf{G}'_{\mathbf{v}}}{\partial \zeta}\right) \quad (1)$$

$$\mathbf{Q}' = \frac{\mathbf{Q}}{J} \qquad \mathbf{E}' = \frac{1}{J}\hat{\mathbf{E}}, \quad (2)$$

$$
\mathbf{Q} = \begin{pmatrix} \bar{\rho} \\ \bar{\rho}\tilde{u} \\ \bar{\rho}\tilde{v} \\ \bar{\rho}\tilde{w} \\ \bar{\rho}\tilde{e} \end{pmatrix}, \ \hat{\mathbf{E}} = \begin{pmatrix} \bar{\rho}\tilde{U} \\ \bar{\rho}\tilde{u}\tilde{U} + \xi_x \tilde{p} \\ \bar{\rho}\tilde{v}\tilde{U} + \xi_y \tilde{p} \\ \bar{\rho}\tilde{w}\tilde{U} + \xi_z \tilde{p} \\ \bar{\rho}\tilde{e}\tilde{U} + \tilde{p}\bar{U} \end{pmatrix},
$$

$$
\tilde{U} = \xi_t + \xi_x \tilde{u} + \xi_y \tilde{v} + \xi_z \tilde{w}, \qquad \bar{U} = \tilde{U} - \xi_t
$$

$$
\mathbf{E_v} = \begin{pmatrix} 0 \\ \bar{\tau}_{xx} - \overline{\rho u'' u''} \\ \bar{\tau}_{xy} - \overline{\rho u'' v''} \\ \bar{\tau}_{xz} - \overline{\rho u'' w''} \\ Q_x \end{pmatrix}
$$

$$\bar{\tau}_{ij} = -\frac{2}{3}\tilde{\mu}\frac{\partial \tilde{u}_k}{\partial x_k}\delta_{ij} + \tilde{\mu}(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}) \tag{3}$$

$$Q_i = \tilde{u}_j(\bar{\tau}_{ij} - \overline{\rho u'' u''}) - (\bar{q}_i + C_p\overline{\rho T'' u_i''}) \tag{4}$$

$$\bar{q}_i = -\frac{\tilde{\mu}}{(\gamma - 1)Pr}\frac{\partial a^2}{\partial x_i} \tag{5}$$

- Molecular viscosity $\tilde{\mu} = \tilde{\mu}(\tilde{T})$
- Total energy:

$$\bar{\rho}\tilde{e} = \frac{\tilde{p}}{(\gamma - 1)} + \frac{1}{2}\bar{\rho}(\tilde{u}^2 + \tilde{v}^2 + \tilde{w}^2) + k \tag{6}$$

- Turbulent: Baldwin-Lomax model

Time Marching Scheme:

Implicit Gauss-Seidel Relaxation, Dual Time Stepping

$$\left[\left(\frac{1}{\Delta\tau}+\frac{1.5}{\Delta t}\right)I-\left(\frac{\partial R}{\partial Q}\right)^{n+1,m}\right]\delta Q^{n+1,m+1}=R^{n+1,m}-\frac{3Q^{n+1,m}-4Q^{n}+Q}{2\Delta t} \tag{7}$$

$$R=-\frac{1}{V}\int_{s}[(F-F_v)\mathbf{i}+(G-G_v)\mathbf{j}+(H-H_v)\mathbf{k}]\cdot d\mathbf{s} \tag{8}$$

Roe's Riemann Solver on Moving Grid System

$$\mathbf{E}'_{i+\frac{1}{2}} = \frac{1}{2}[\mathbf{E}''(\mathbf{Q_L})+\mathbf{E}''(\mathbf{Q_R})+\mathbf{Q_L}\xi_{t\,L}+\mathbf{Q_R}\xi_{t\,R}-|\tilde{\mathbf{A}}|(\mathbf{Q_R}-\mathbf{Q_L})]_{i+\frac{1}{2}} \tag{9}$$

$$\tilde{\mathbf{A}} = \tilde{\mathbf{T}}\tilde{\mathbf{\Lambda}}\tilde{\mathbf{T}}^{-1} \tag{10}$$

$$(\tilde{U}+\tilde{C}, \tilde{U}-\tilde{C}, \tilde{U}, \tilde{U}, \tilde{U}) \tag{11}$$

$$\tilde{U} = \tilde{\xi}_t + \xi_x\tilde{u} + \xi_y\tilde{v} + \xi_z\tilde{w} \tag{12}$$

$$\tilde{C} = \tilde{c}\sqrt{\xi_x^2+\xi_y^2+\xi_z^2} \tag{13}$$

$$\tilde{\xi}_t = (\xi_{t\,L} + \xi_{t\,R}\sqrt{\rho_R/\rho_L})/(1+\sqrt{\rho_R/\rho_L}) \tag{14}$$

Boundary Conditions

- Upstream: All variables specified except pressure extrapolated from interior

- Downstream: All variables extrapolated except pressure specified

- Solid wall boundary conditions: Non-slip condition

$$u_0 = 2\dot{x}_b - u_1, \qquad v_0 = 2\dot{y}_b - v_1 \tag{15}$$

and adiabatic and the inviscid normal momentum equation

$$\frac{\partial T}{\partial \eta} = 0, \qquad \frac{\partial p}{\partial \eta} = -\left(\frac{\rho}{\eta_x^2 + \eta_y^2}\right)(\eta_x \ddot{x}_b + \eta_y \ddot{y}_b) \tag{16}$$

Geometric Conservation Law

$$\mathbf{S} = \mathbf{Q}\left[\frac{\partial J^{-1}}{\partial t} + \left(\frac{\xi_t}{J}\right)_\xi + \left(\frac{\eta_t}{J}\right)_\eta + \left(\frac{\zeta_t}{J}\right)_\zeta\right] \tag{17}$$

$$\mathbf{S}^{n+1} = \mathbf{S}^n + \frac{\partial \mathbf{S}}{\partial \mathbf{Q}}\Delta \mathbf{Q}^{n+1} \tag{18}$$

Structural Model

Governing equation:

$$\mathbf{M}\frac{d^2\mathbf{u}}{dt^2} + \mathbf{C}\frac{d\mathbf{u}}{dt} + \mathbf{K}\mathbf{u} = \mathbf{f} \tag{19}$$

$$\mathbf{u} = \begin{pmatrix} \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_i \\ \vdots \\ \mathbf{u}_N \end{pmatrix}, \ \mathbf{f} = \begin{pmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_i \\ \vdots \\ \mathbf{f}_N \end{pmatrix}, \ \mathbf{u}_i = \begin{pmatrix} \mathbf{u}_{ix} \\ \mathbf{u}_{iy} \\ \mathbf{u}_{iz} \end{pmatrix}, \ \mathbf{f}_i = \begin{pmatrix} \mathbf{f}_{ix} \\ \mathbf{f}_{iy} \\ \mathbf{f}_{iz} \end{pmatrix}.$$

Modal Approach

$$\mathbf{u}(t) = \sum_j a_j(t)\phi_j = \mathbf{\Phi a} \tag{20}$$

Mode shape matrix: $\mathbf{\Phi} = [\phi_1, \cdots, \phi_j, \cdots, \phi_{3N}]$.

Modal Coordinates: $\mathbf{a} = [a_1, a_2, a_3, \cdots]^T$

$$\frac{d^2 a_j}{dt^2} + 2\zeta_j\omega_j\frac{da_j}{dt} + \omega_j^2 a_j = \frac{\phi_j^T \mathbf{f}}{m_j} \tag{21}$$

State form:

$$[\mathbf{M}]\frac{\partial\{\mathbf{S}\}}{\partial t} + [\mathbf{K}]\{\mathbf{S}\} = \mathbf{q} \tag{22}$$

where

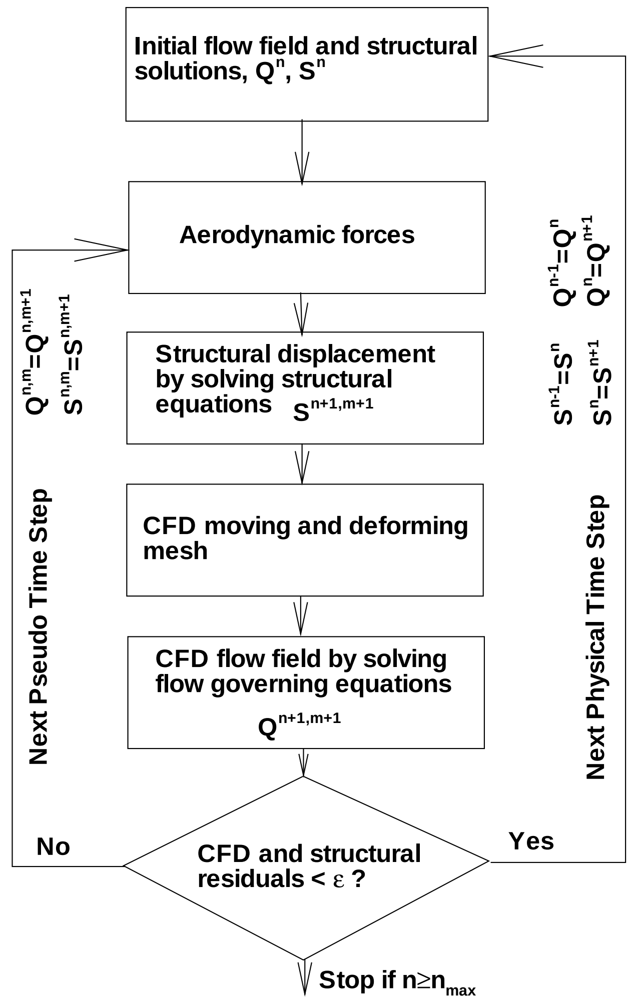
$$\mathbf{S} = \begin{pmatrix} a_j \\ \dot{a}_j \end{pmatrix}, \; \mathbf{M} = [I], \; \mathbf{K} = \begin{pmatrix} 0 & -1 \\ \left(\frac{\omega_j}{\omega_\alpha}\right)^2 & 2\zeta\left(\frac{\omega_j}{\omega_\alpha}\right) \end{pmatrix},$$

$$\mathbf{q} = \begin{pmatrix} 0 \\ \phi_j^{*T}\mathbf{f}^* V^* \left(\frac{b_s}{L}\right)^2 \frac{\bar{m}}{v^*} \end{pmatrix}.$$

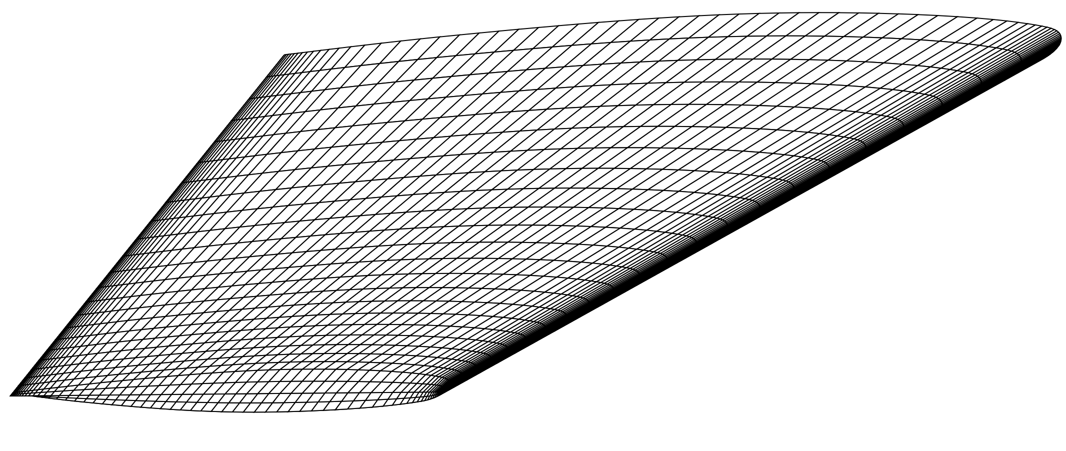
Time marching: same as flow solver

$$\left(\frac{1}{\Delta\tau}\mathbf{I} + \frac{1.5}{\Delta t}\mathbf{M} + \mathbf{K}\right)\delta S^{n+1,m+1} = -\mathbf{M}\frac{3\mathbf{S}^{n+1,m} - 4\mathbf{S}^n + \mathbf{S}^{n-1}}{2\Delta t} - \mathbf{K}\mathbf{S}^{n+1,m} + \mathbf{q} \tag{23}$$

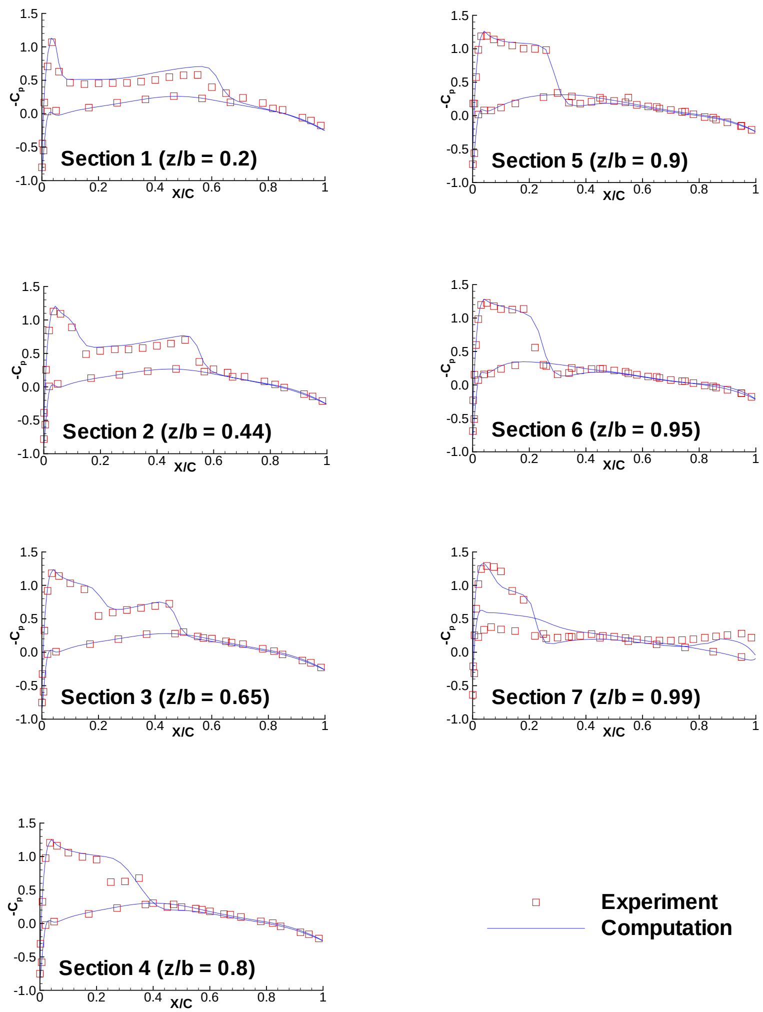
Fully Coupled Fluid-Structural Interaction Procedure



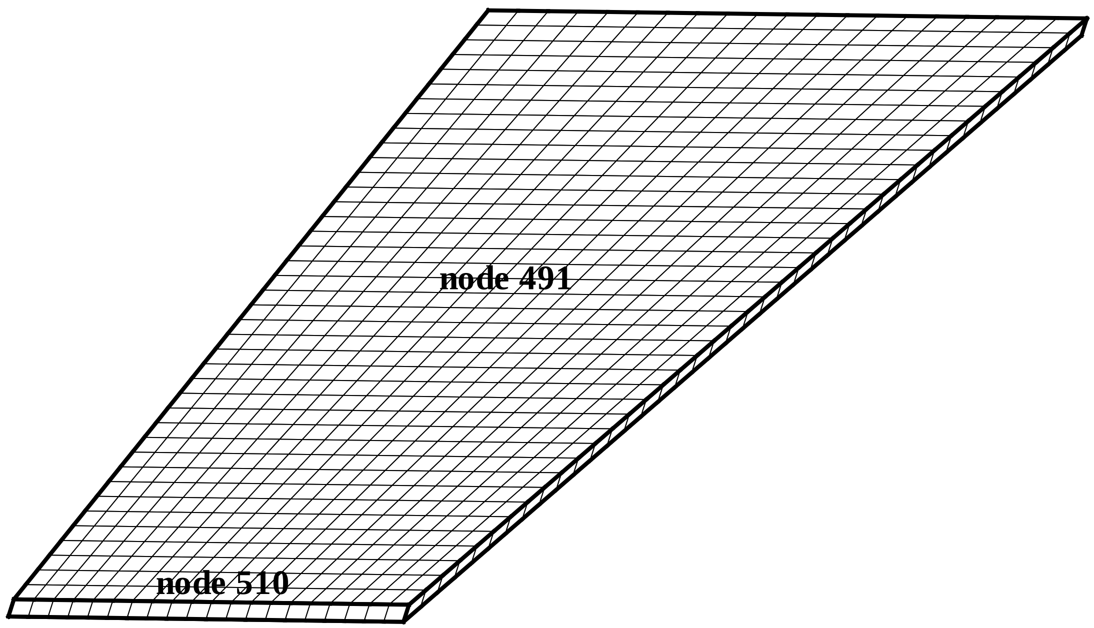
Results: ONERA M6 Wing Mesh M=0.84, Re=19.7×10^6



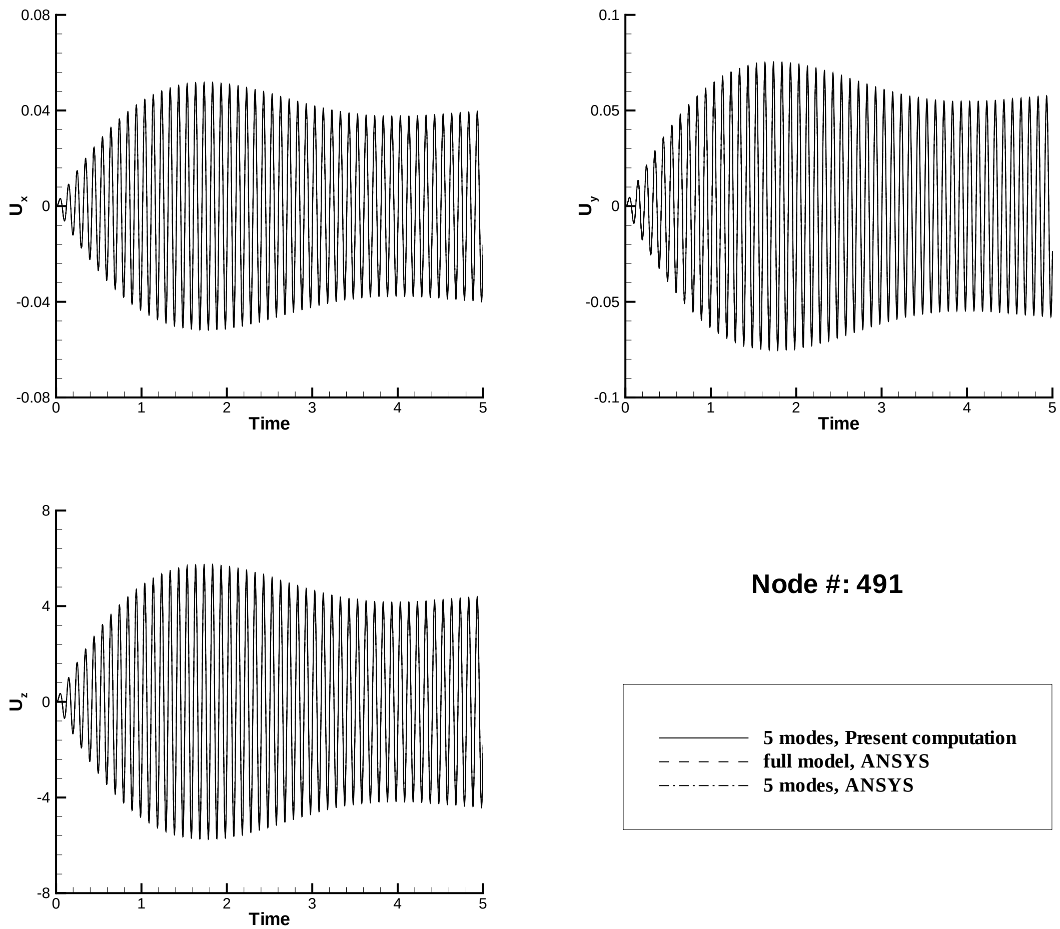
ONERA M6 Wing Surface Pressure



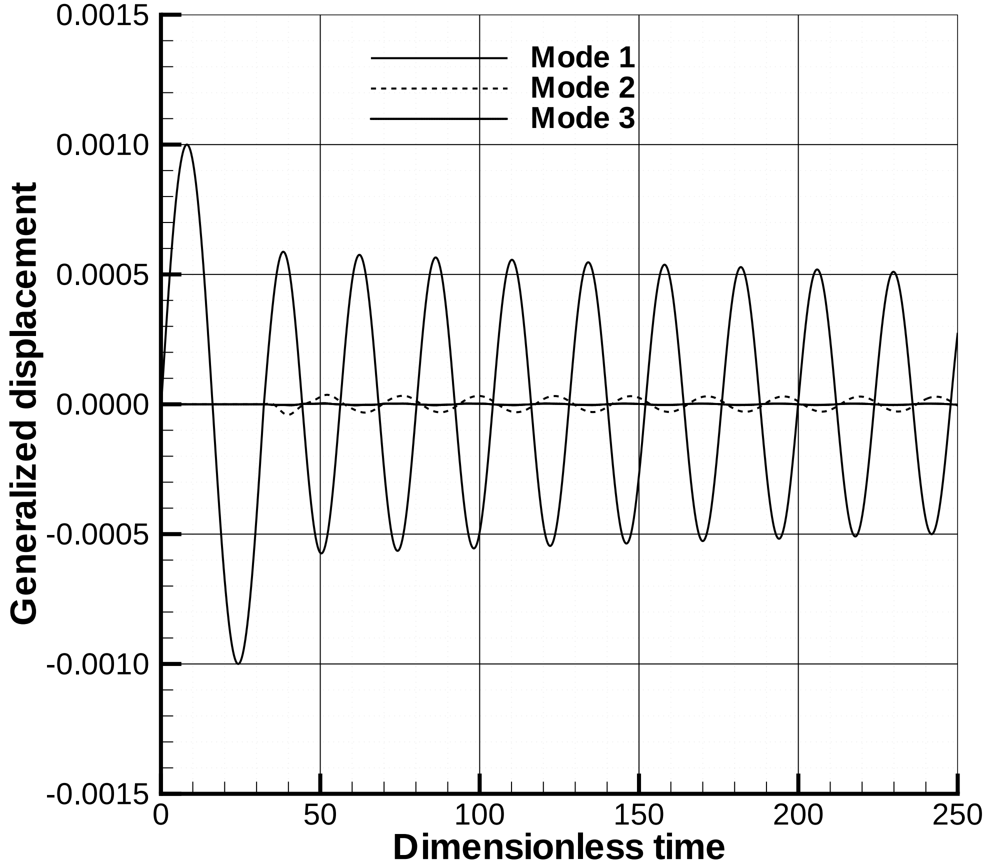
Structural Solver Validation: Plate Wing



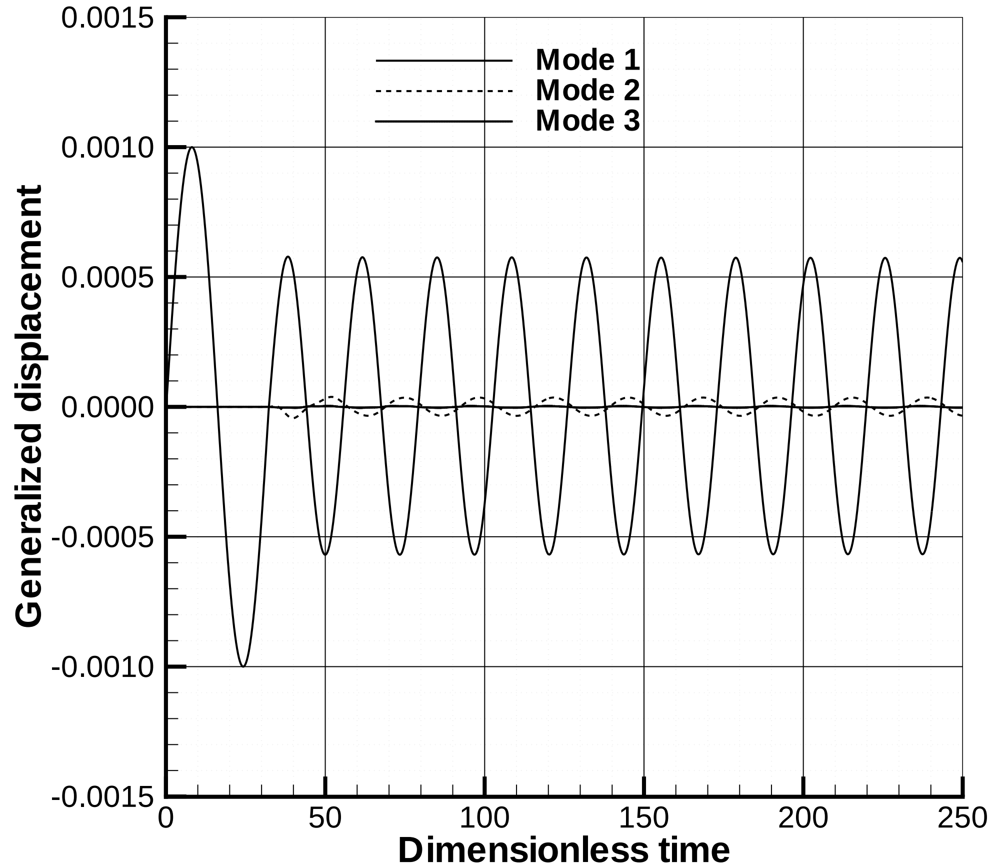
Structural Solver Validation: Plate Wing



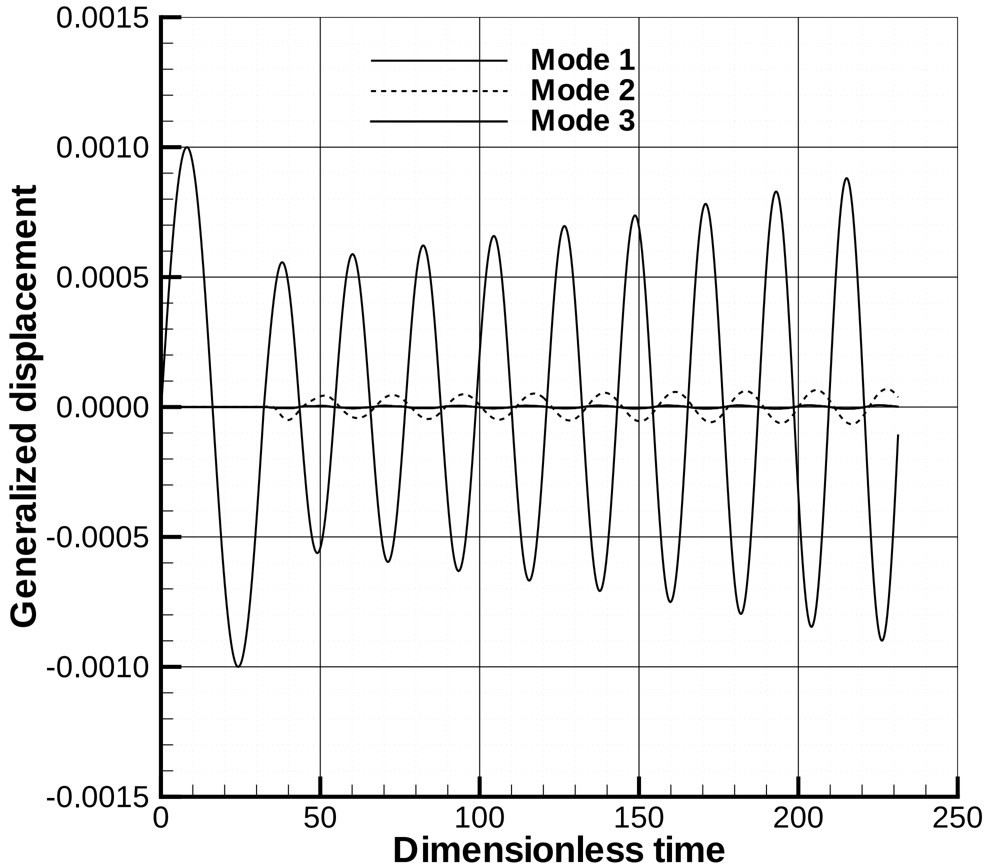
AGARD Wing 445.6, Damped response $M_\infty = 0.96$ and V^* $= 0.28$, 1st 5 Modes



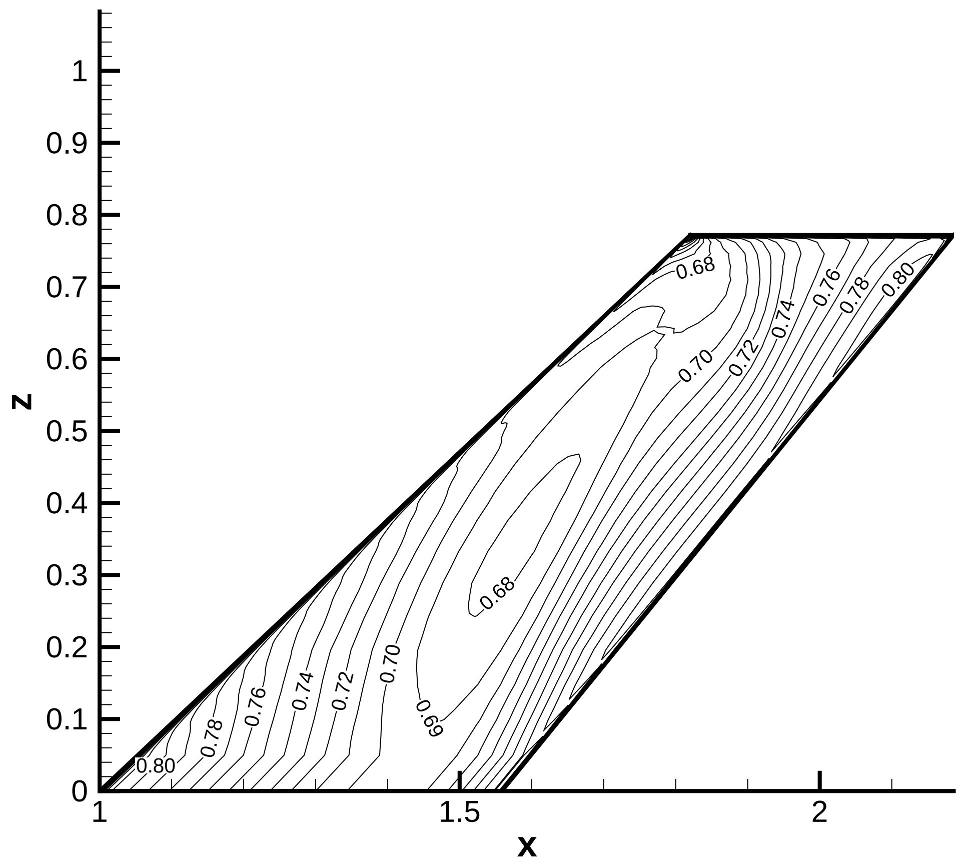
AGARD Wing 445.6, Neutrally stable response. $M_\infty =$ 0.96 and $V^* = 0.29$, 1st 5 Modes



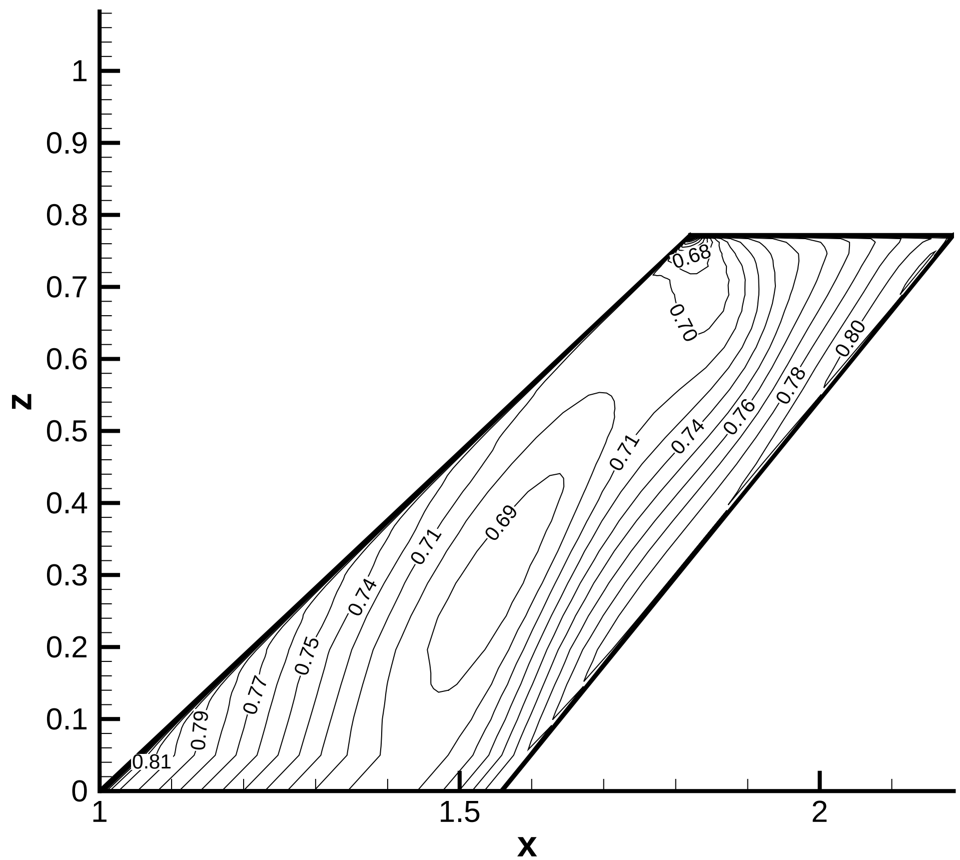
AGARD Wing 445.6, Diverging response $M_{\infty} = 0.96$ and V^* $= 0.315$, 1st 5 Modes



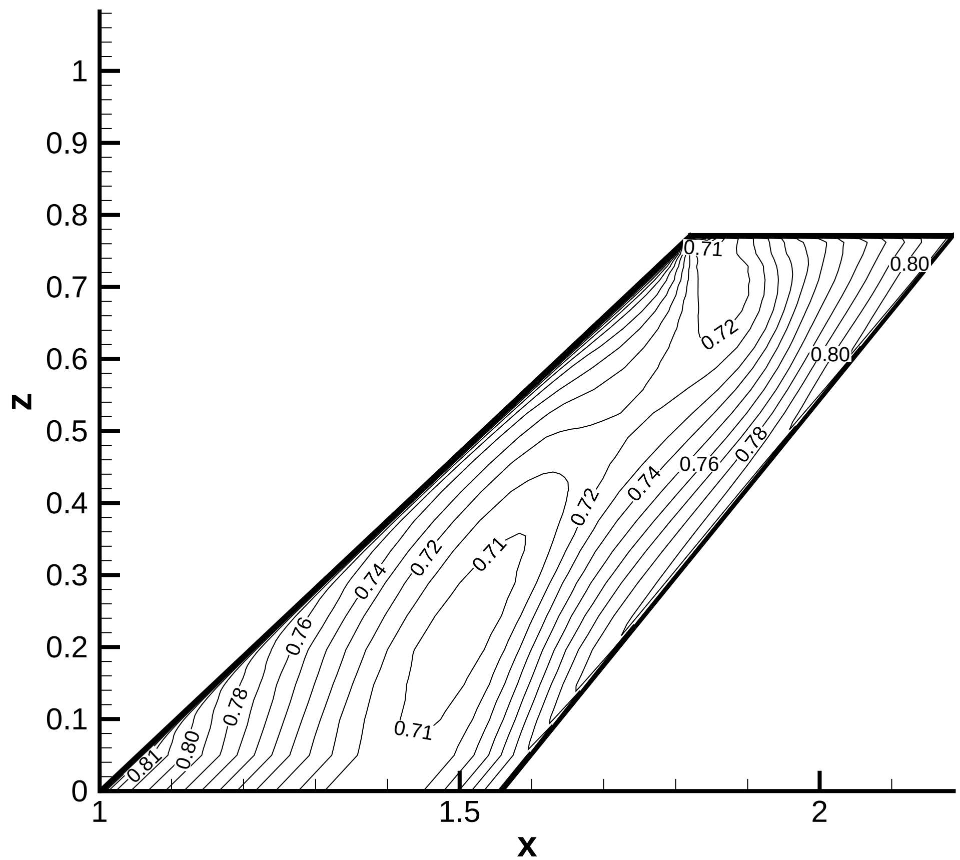
Pressure Contours, Uppermost position $M_\infty = 0.96$ and V^* $= 0.29$



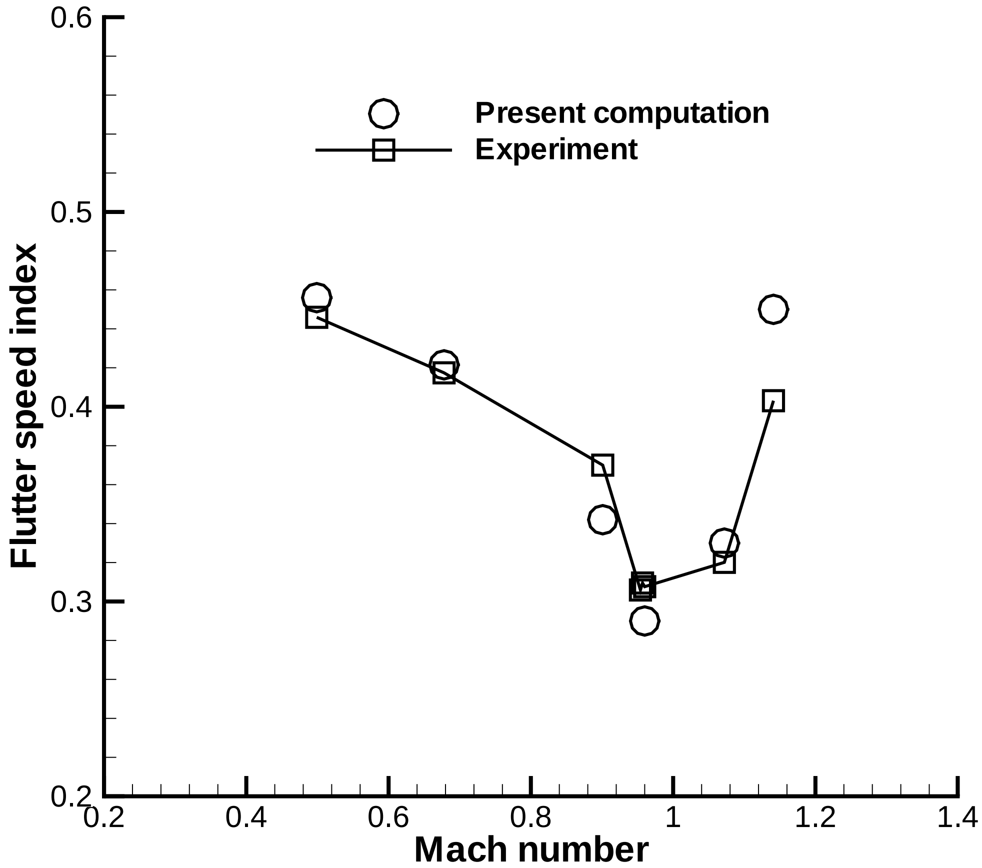
Pressure Contours, Neutral position $M_\infty = 0.96$ and $V^* = 0.29$



Pressure Contours, Lowermost position $M_\infty = 0.96$ and V^* $= 0.29$



Computed AGARD Wing 445.6 Flutter Boundary



Conclusions

• A fully coupled 3D fluid-structural interaction methodology for flutter prediction is developed.

• A dual time stepping implicit unfactored Gauss-Seidel iteration with Roe scheme is employed.

• The structural response using modal approach with 1st 5 modes agrees excellently with finite element solution.

• The predicted AGARD Wing 445.6 flutter boundary agrees well with experiment.