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Parallel Computation of Forced Vibration for A Compressor Cascade

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Background

- Turbomachinery Blade Flutter Important
- Mistuned Rotor Require Full Annulus Simulation

Objective

- Develop and Validate CFD Code for Multi-blade Passage Vibration

Numerical Strategy

- 3D Time Accurate RANS
- Dual Time Stepping, Implicit Gauss-Seidel Iteration
- Low Diffusion Zha E-CUSP2 Scheme
- 2nd Order Accuracy in Space and Time
- Baldwin-Lomax Turbulence Model
- Enforce Geometry Conservation Law
- Validate with NASA Flutter Cascade Experiment

Governing Equations: 3D RANS

$$\frac{\partial \mathbf{Q}'}{\partial \tau} + \frac{\partial \mathbf{Q}'}{\partial t} + \frac{\partial \mathbf{E}'}{\partial \xi} + \frac{\partial \mathbf{F}'}{\partial \eta} + \frac{\partial \mathbf{G}'}{\partial \zeta} = \frac{1}{Re}\left(\frac{\partial \mathbf{E}'_{\mathbf{v}}}{\partial \xi} + \frac{\partial \mathbf{F}'_{\mathbf{v}}}{\partial \eta} + \frac{\partial \mathbf{G}'_{\mathbf{v}}}{\partial \zeta}\right) \quad (1)$$

$$\mathbf{Q}' = \frac{\mathbf{Q}}{J} \qquad \mathbf{E}' = \frac{1}{J}\hat{\mathbf{E}}, \quad (2)$$

$$\mathbf{Q} = \begin{pmatrix} \bar{\rho} \\ \bar{\rho}\tilde{u} \\ \bar{\rho}\tilde{v} \\ \bar{\rho}\tilde{w} \\ \bar{\rho}\tilde{e} \end{pmatrix}, \hat{\mathbf{E}} = \begin{pmatrix} \bar{\rho}\tilde{U} \\ \bar{\rho}\tilde{u}\tilde{U} + \xi_x\tilde{p} \\ \bar{\rho}\tilde{v}\tilde{U} + \xi_y\tilde{p} \\ \bar{\rho}\tilde{w}\tilde{U} + \xi_z\tilde{p} \\ \bar{\rho}\tilde{e}\tilde{U} + \tilde{p}\bar{U} \end{pmatrix},$$

$$\tilde{U} = \xi_t + \xi_x\tilde{u} + \xi_y\tilde{v} + \xi_z\tilde{w}, \qquad \bar{U} = \tilde{U} - \xi_t$$

Time Marching Scheme:

Implicit Gauss-Seidel Relaxation, Dual Time Stepping

$$\left[\left(\frac{1}{\Delta\tau} + \frac{1.5}{\Delta t}\right) I - \left(\frac{\partial R}{\partial Q}\right)^{n+1,m}\right] \delta Q^{n+1,m+1} = R^{n+1,m} - \frac{3Q^{n+1,m} - 4Q^{n} + Q^{\cdots}}{2\Delta t} \tag{3}$$

Upwind Schemes Implemented: Roe, van Leer, Liou's AUSM+, New E-CUSP2

The New E-CUSP Scheme with High Efficiency Low Diffusion

$$\hat{\mathbf{E}} = \hat{\mathbf{A}}\mathbf{Q} = \hat{\mathbf{T}}\hat{\mathbf{\Lambda}}\hat{\mathbf{T}}^{-1}\mathbf{Q} \tag{4}$$

$$\hat{\mathbf{\Lambda}} = \begin{pmatrix} \tilde{U}-\tilde{C} & 0 & 0 & 0 & 0 \\ 0 & \tilde{U} & 0 & 0 & 0 \\ 0 & 0 & \tilde{U} & 0 & 0 \\ 0 & 0 & 0 & \tilde{U} & 0 \\ 0 & 0 & 0 & 0 & \tilde{U}+\tilde{C} \end{pmatrix} = \tilde{U}[\mathbf{I}] + \begin{pmatrix} -\tilde{C} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{C} \end{pmatrix} \tag{5}$$

$$
\hat{\mathbf{E}} = \{\tilde{U}[\mathbf{I}] + \begin{pmatrix} -\tilde{C} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{C} \end{pmatrix} \}\mathbf{Q}
$$

$$
= \hat{\mathbf{E}}^c + \hat{\mathbf{E}}^p = \begin{pmatrix} \bar{\rho}\tilde{U} \\ \bar{\rho}\tilde{u}\tilde{U} \\ \bar{\rho}\tilde{v}\tilde{U} \\ \bar{\rho}\tilde{w}\tilde{U} \\ \bar{\rho}\tilde{e}\tilde{U} \end{pmatrix} + \begin{pmatrix} 0 \\ \xi_x\tilde{p} \\ \xi_y\tilde{p} \\ \xi_z\tilde{p} \\ \tilde{p}\bar{U} \end{pmatrix} \tag{6}
$$

For subsonic flow, $M < 1$:

$$\hat{\mathbf{E}}_{\frac{1}{2}} = \frac{1}{2}[(\bar{\rho}\tilde{U})_{\frac{1}{2}}(\mathbf{q^c}_L + \mathbf{q^c}_R) - |\bar{\rho}\tilde{U}|_{\frac{1}{2}}(\mathbf{q^c}_R - \mathbf{q^c}_L)]$$
$$+ \begin{pmatrix} 0 \\ \mathcal{P}^+\tilde{p}\xi_x \\ \mathcal{P}^+\tilde{p}\xi_y \\ \mathcal{P}^+\tilde{p}\xi_z \\ \frac{1}{2}\tilde{p}(\bar{U} + \bar{C}_{\frac{1}{2}}) \end{pmatrix}_L + \begin{pmatrix} 0 \\ \mathcal{P}^-\tilde{p}\xi_x \\ \mathcal{P}^-\tilde{p}\xi_y \\ \mathcal{P}^-\tilde{p}\xi_z \\ \frac{1}{2}\tilde{p}(\bar{U} - \bar{C}_{\frac{1}{2}}) \end{pmatrix}_R \tag{7}$$

where

$$(\bar{\rho}\tilde{U})_{\frac{1}{2}} = (\bar{\rho}_L\tilde{U}_L^+ + \bar{\rho}_R\tilde{U}_R^-) \tag{8}$$

$$\mathbf{q^c} = \begin{pmatrix} 1 \\ \tilde{u} \\ \tilde{v} \\ \tilde{w} \\ \tilde{e} \end{pmatrix} \tag{9}$$

$$\tilde{C}_{\frac{1}{2}} = \frac{1}{2}(\tilde{C}_L + \tilde{C}_R) \tag{10}$$

$$\tilde{M}_L = \frac{\tilde{U}_L}{\tilde{C}_{\frac{1}{2}}}, \qquad \tilde{M}_R = \frac{\tilde{U}_R}{\tilde{C}_{\frac{1}{2}}} \tag{11}$$

$$\tilde{U}_L^+ = \tilde{C}_{\frac{1}{2}}\{\frac{\tilde{M}_L + |\tilde{M}_L|}{2} + \alpha_L[\frac{1}{4}(\tilde{M}_L + 1)^2 - \frac{\tilde{M}_L + |\tilde{M}_L|}{2}]\} \tag{12}$$

$$\tilde{U}_R^- = \tilde{C}_{\frac{1}{2}}\{\frac{\tilde{M}_R - |\tilde{M}_R|}{2} + \alpha_R[-\frac{1}{4}(\tilde{M}_R - 1)^2 - \frac{\tilde{M}_R - |\tilde{M}_R|}{2}]\} \tag{13}$$

$$\alpha_L = \frac{2(\tilde{p}/\bar{\rho})_L}{(\tilde{p}/\bar{\rho})_L + (\tilde{p}/\bar{\rho})_R}, \qquad \alpha_R = \frac{2(\tilde{p}/\bar{\rho})_R}{(\tilde{p}/\bar{\rho})_L + (\tilde{p}/\bar{\rho})_R} \tag{14}$$

$$\mathcal{P}^\pm = \frac{1}{4}(\tilde{M} \pm 1)^2(2 \mp \tilde{M}) \pm \alpha\tilde{M}(\tilde{M}^2 - 1)^2, \qquad \alpha = \frac{3}{16} \tag{15}$$

$$\bar{C} = \tilde{C} - \xi_t \tag{16}$$

$$\bar{C}_{\frac{1}{2}} = \frac{1}{2}(\bar{C}_L + \bar{C}_R) \tag{17}$$

For Energy Eq.

$$\alpha_L = \frac{2(\tilde{H}/\bar{\rho})_L}{(\tilde{H}/\bar{\rho})_L + (\tilde{H}/\bar{\rho})_R}, \qquad \alpha_R = \frac{2(\tilde{H}/\bar{\rho})_R}{(\tilde{H}/\bar{\rho})_L + (\tilde{H}/\bar{\rho})_R} \tag{18}$$

For supersonic flow,

when $\tilde{U}_L \geq \tilde{C}$, $\hat{\mathbf{E}}_{\frac{1}{2}} = \hat{\mathbf{E}}_L$

when $\tilde{U}_R \leq -\tilde{C}$, $\hat{\mathbf{E}}_{\frac{1}{2}} = \hat{\mathbf{E}}_R$

Boundary Conditions

No slip adiabatic wall condition:

$$u_o = 2u_w - u_i \tag{19}$$

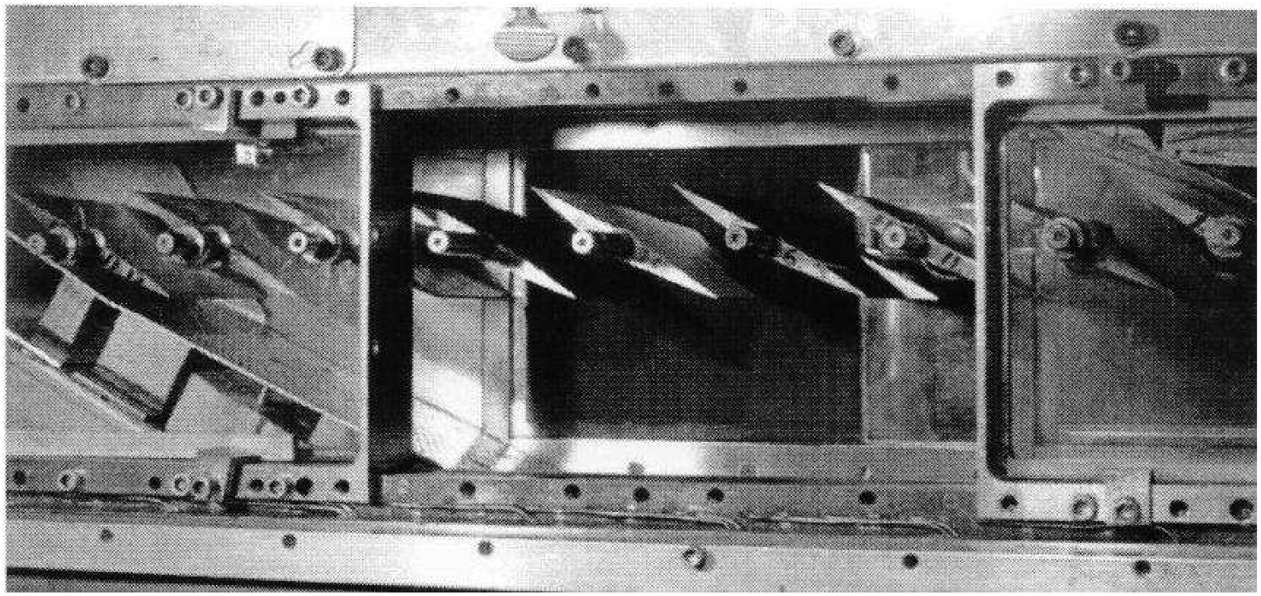
$$\frac{\partial T}{\partial \eta} = 0 \tag{20}$$

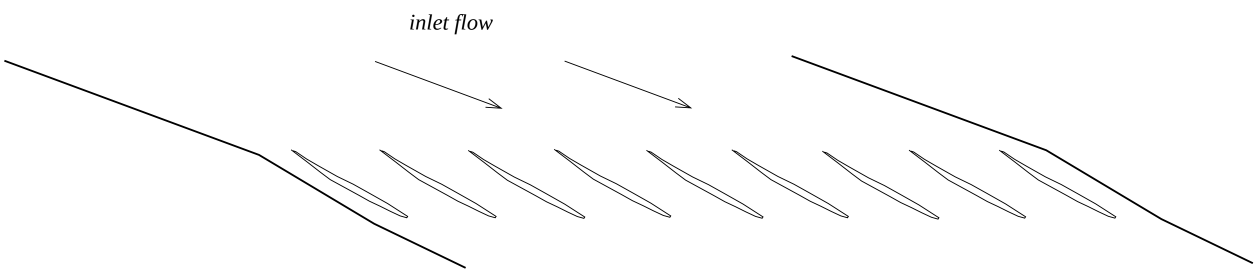
$$\frac{\partial p}{\partial \eta} = -\left(\frac{\rho}{\eta_x^2 + \eta_y^2}\right)(\eta_x \dot{u}_w + \eta_y \dot{v}_w) \tag{21}$$

Upstream: Constant P_t, T_t, Flow Angle

Downstream: Constant static pressure

NASA transonic flutter cascade





Forced Oscillation, IBPA=180°

$$\alpha^n(t) = \alpha_0 + \hat{\alpha} Re\left[\exp\left(i\left(\omega t + n\beta\right)\right)\right] \qquad (22)$$

Parameters

Reduced Frequency:

$$k_c = \frac{\omega C}{U_\infty} \tag{23}$$

Fourier Transformed Unsteady Pressure Coefficient.

$$C_p\left(x\right) = \frac{p_1\left(x\right)}{\rho_\infty U_\infty^2 \hat{\alpha}} \tag{24}$$

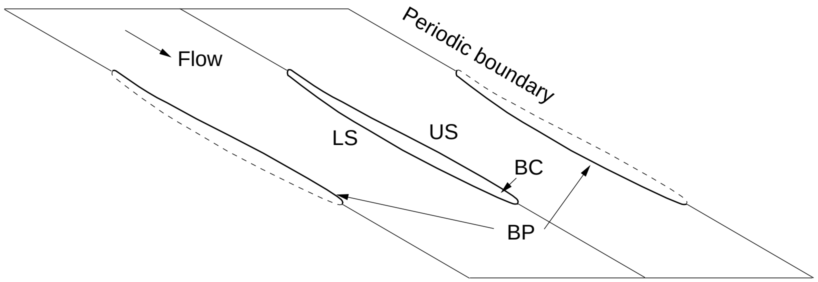
Unsteady Moment Coefficient

$$C_m\left(t\right) = \frac{-\displaystyle\int \mathbf{r} \times p\left(x\right) \mathrm{d}\mathbf{s}}{\frac{1}{2}\rho_\infty U_\infty^2 \hat{\alpha}} \tag{25}$$

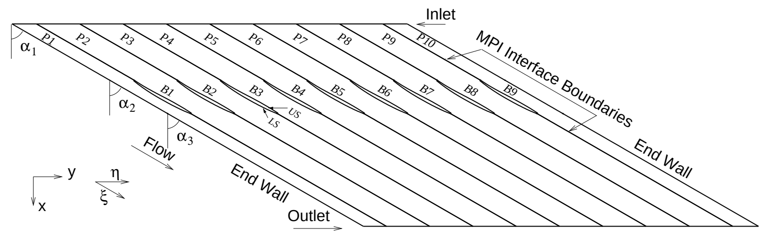
Aerodynamic damping coefficient:

$$\Xi = -Im\left(C_m\right) \tag{26}$$

Results: Computation Domain



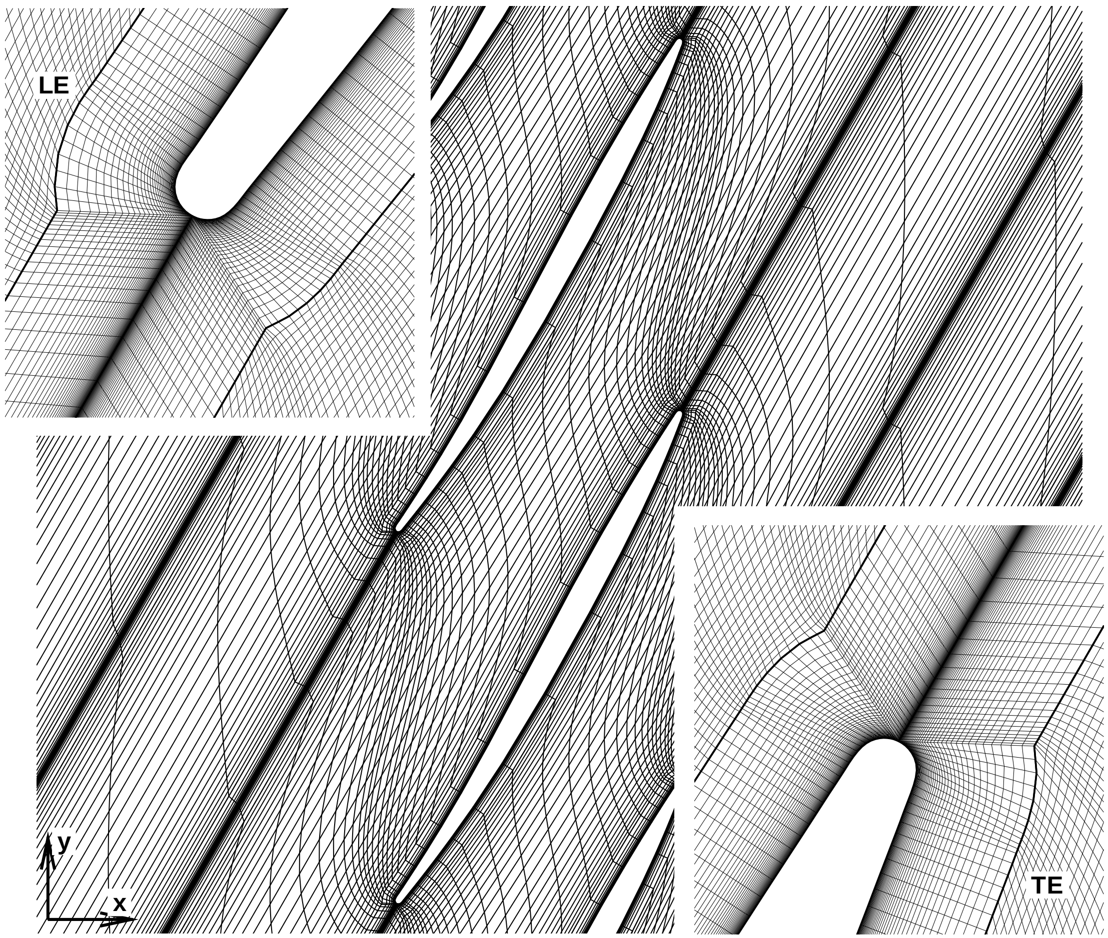
two passages



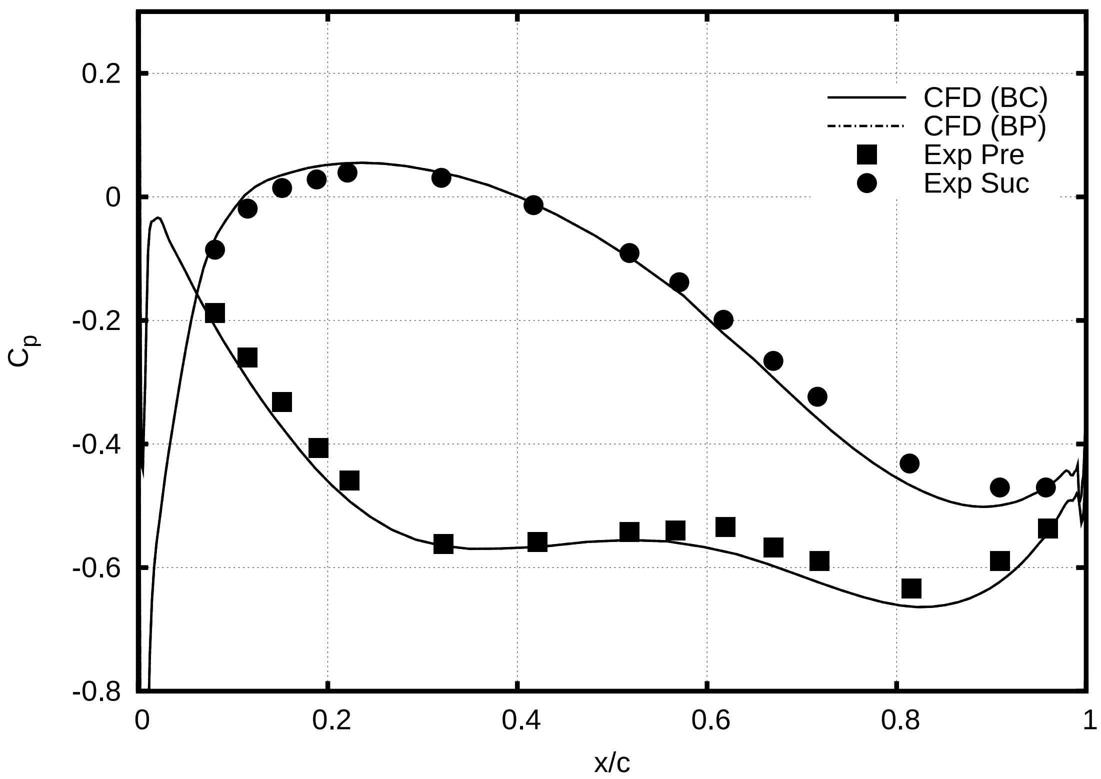
full scale multi-passages with wind tunnel walls

M=0.5, Re= 9×10^5, Incidence = 1 °, $K_c = 0.4, 0.8, 1.2$

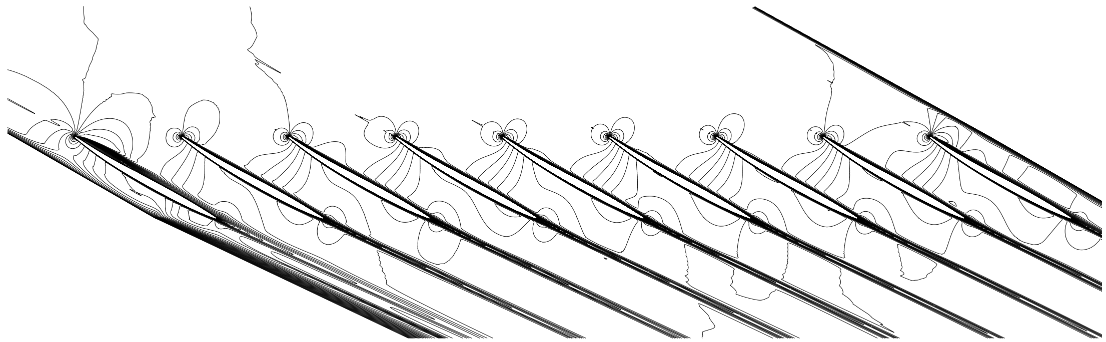
Results: Mesh



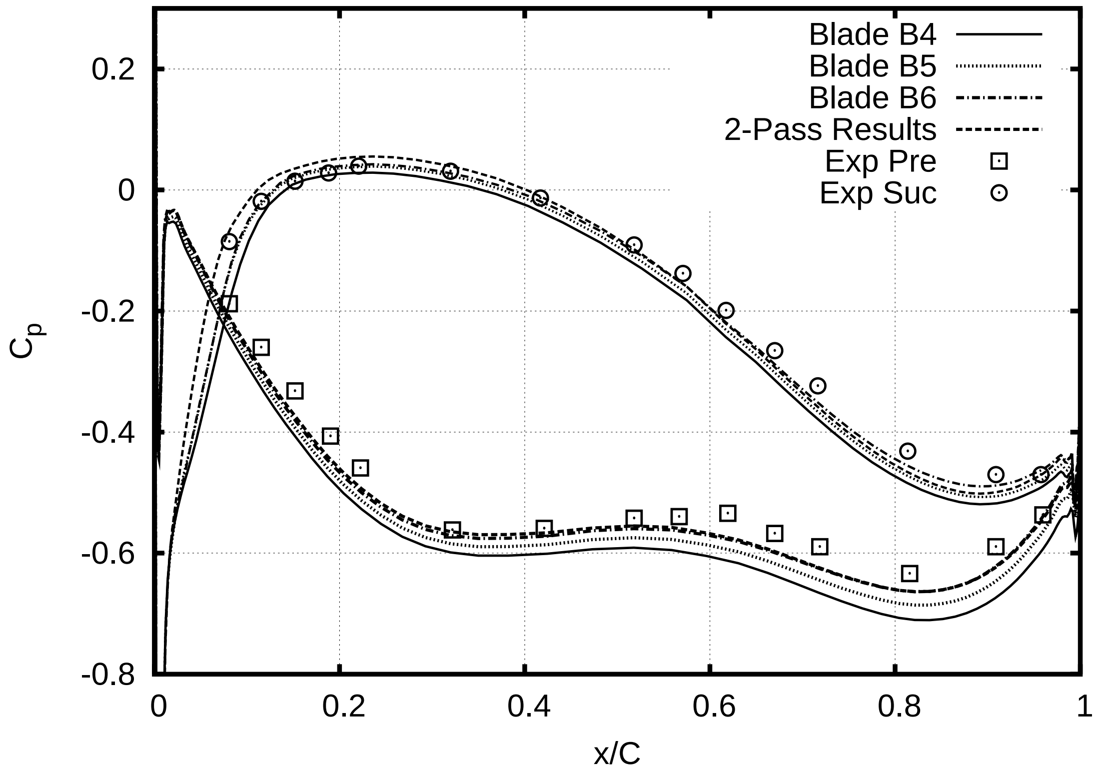
Unsteady pressure oscillation, 2 passages, $K_c = 0.8$



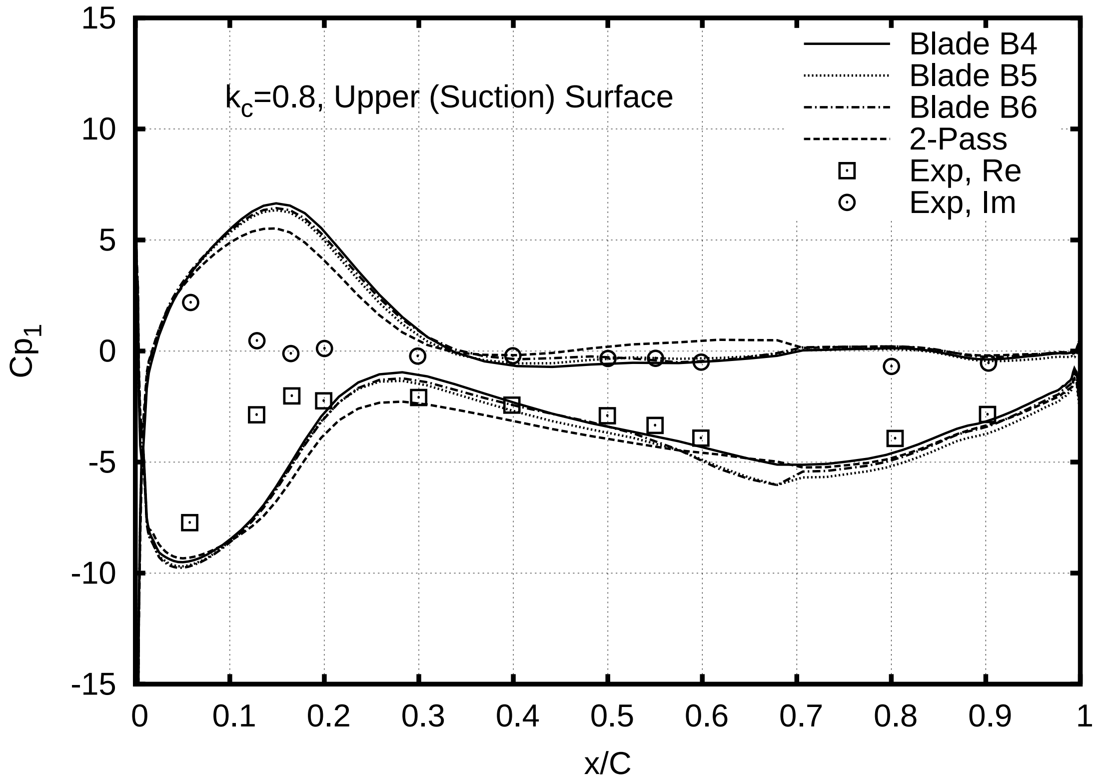
Steady state Mach number contours, Multi-passages



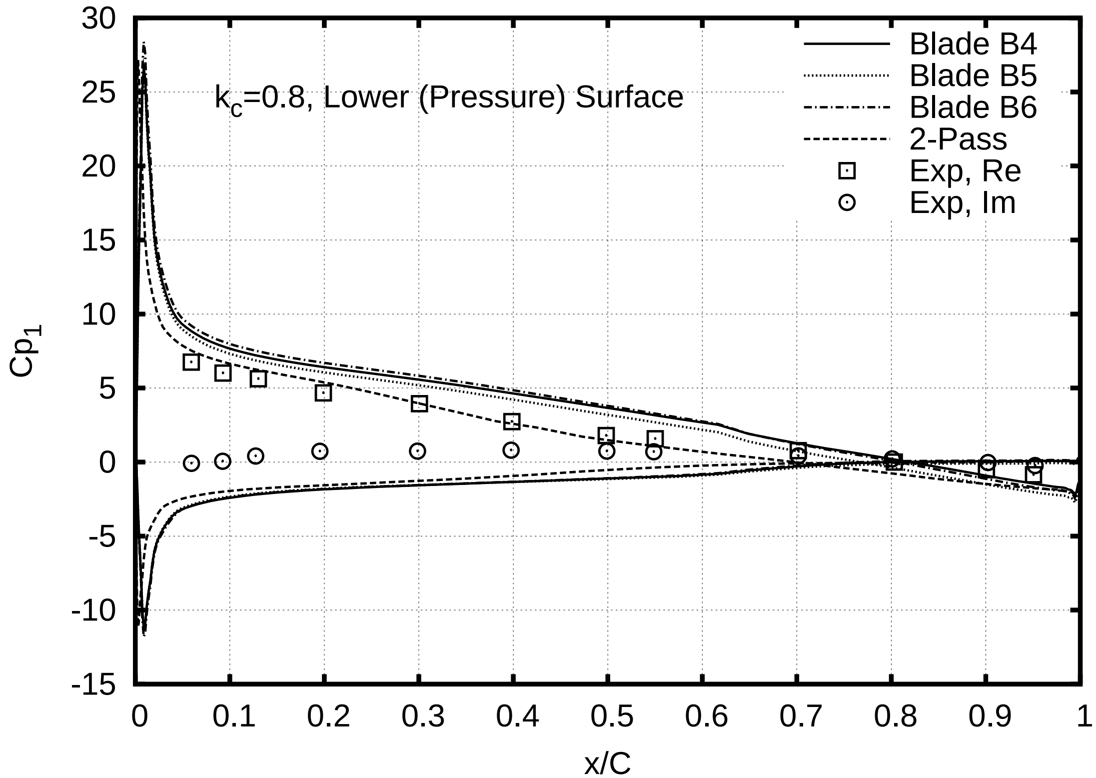
Steady state surface pressure



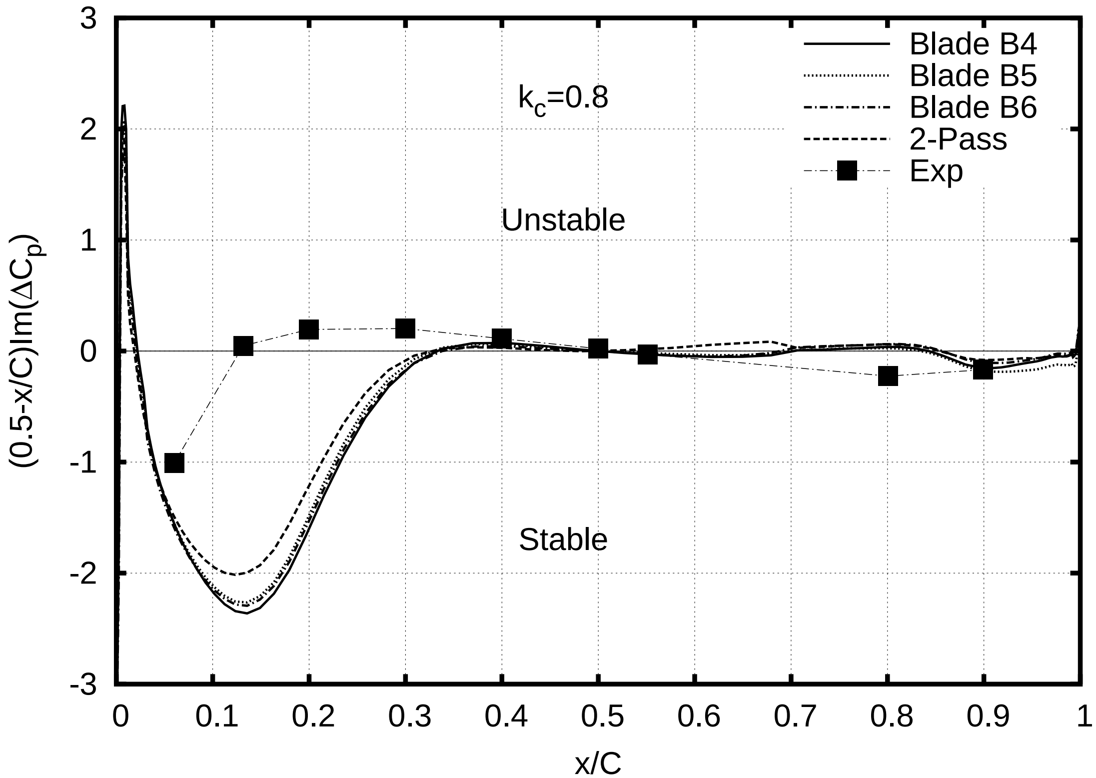
Unsteady surface pressure, suction surface, $K_c = 0.8$



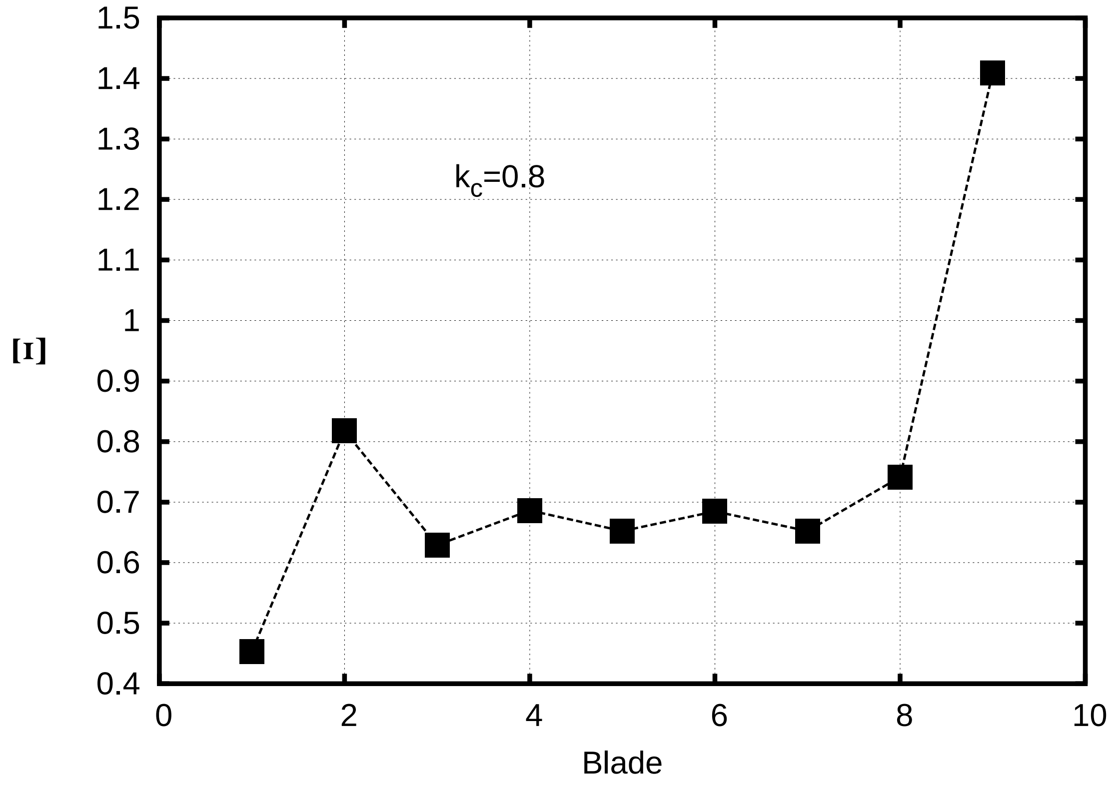
Unsteady surface pressure, pressure surface, $K_c = 0.8$



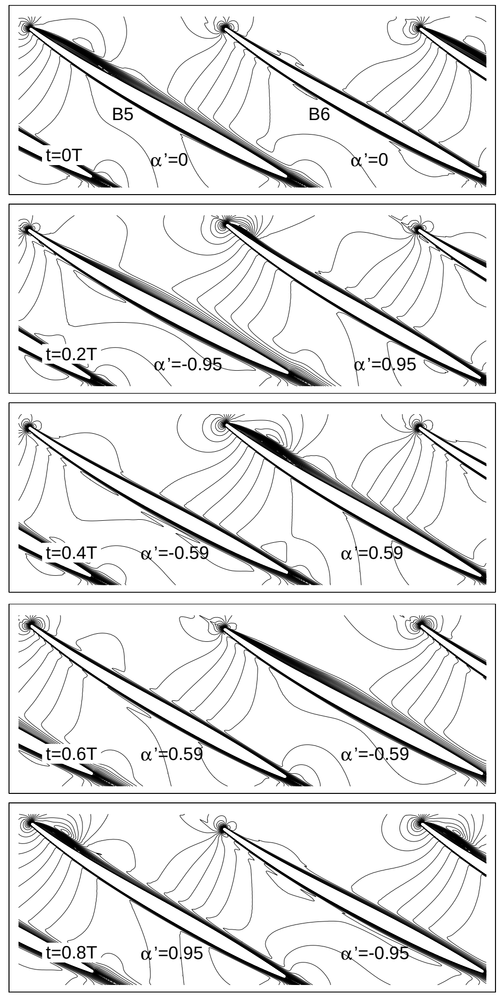
Local stability analysis, $k_c = 0.8$



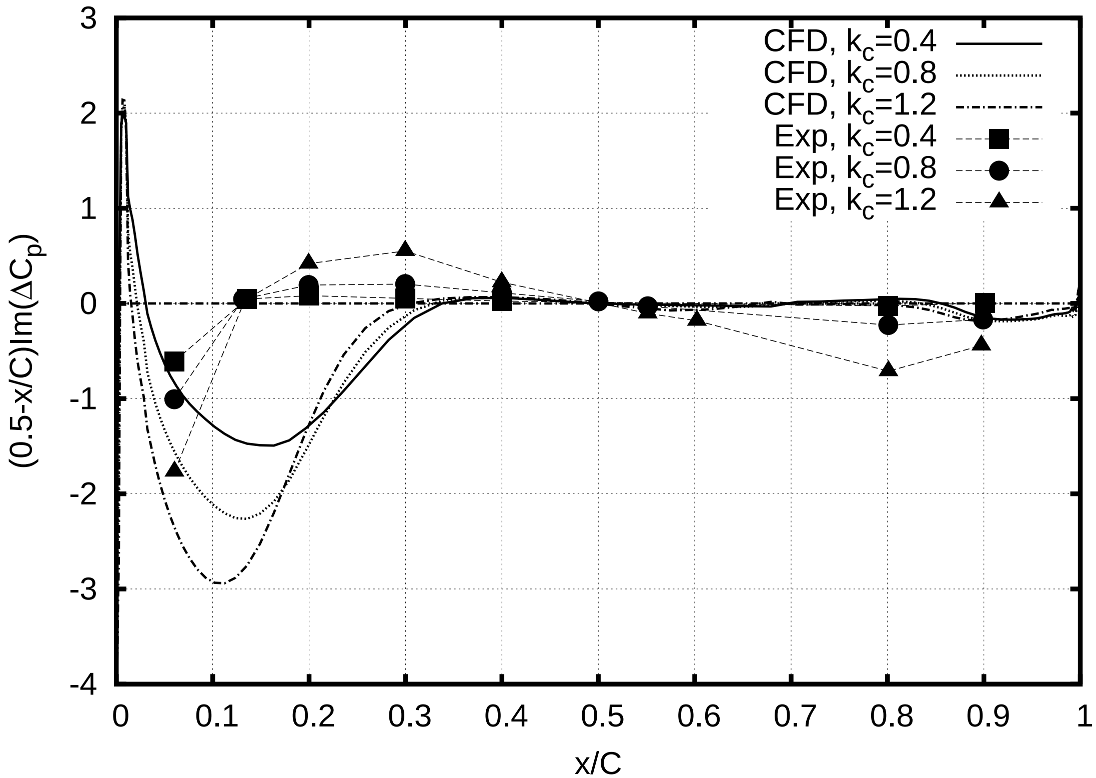
Damping coefficient distribution, $k_c = 0.8$



Unsteady Mach number contours, $k_c = 0.8$



Local Stability at different k_c



Conclusions

- A parallel computation methodology for multi-passage fluid-structural interaction of blade cascade is developed and validated with NASA Flutter Cascade experiment.
- The steady state surface pressure agrees well with experiment.
- The unsteady surface pressure agrees reasonably well with experiment, with large discrepancy in the front part of the blade.
- The trend of blade local stability agrees well with experiment, the quantity is over-predicted in the front part of the blade.
- The aerodynamic damping is increased with the increased vibration frequency.
- The wind tunnel end wall has effect on the blade flow in the middle blade of the cascade.
- The detailed unsteady vortex shedding phenomenon is captured.